ANALYSIS AND PERFORMANCE EVALUATION OF COORDINATED TRANSACTION SCHEDULING

BY

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THESIS

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ABSTRACT

In this thesis, we focus on *coordinated transaction scheduling* (CTS)—an interchange evaluation methodology that is deployed by the New York Independent System Operator ($NYISO$) and Independent System Operator-New England (ISO-NE) since December 15, 2015. The analysis and the quantification of the performance of any interchange evaluation methodology require the explicit representation of the physical aspects, the economic aspects, the steps of the coordination procedure, and all the interactions among them. In order to consider the required representation and tools in a unified structure, we construct a framework. This framework is general and comprehensive, and can be used as a consistent basis that allows the side-by-side comparison of any two interchange evaluation methodologies.

We tailor this framework for CTS, and construct appropriate models of the power system assets and the economics of the two interconnected IGOs, as well as their interactions. We provide an analytical underpinning of the procedural steps of CTS, all the while taking considerable care to maintain

the unshared information confidentiality as the private information of each *IGO*. We perform an assessment of the duration of each procedural step, and identify the binding constraints in the analysis of shorter coordination periods. We use the actual real-time market prices at Sandy Pond and Roseton busses, and evaluate the dependence of the interchange on the coordination period duration. In order to illustrate the execution of the procedural steps of *CTS*, we use various data sets, and we evaluate the interface exchange and the total payments for each data set. We conduct sensitivity analyses, and examine the dependence of the interface exchange and the total payments on a change in the internal offers or the interface offers.

To my family

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CHAPTER 1

INTRODUCTION

In this chapter, we describe the nature of the problems considered in this thesis, review the relevant literature and discuss the scope and the key contributions of the thesis. We provide the nature of the problems discussed in this thesis in Section 1.1. In Section 1.2, we review the state of the art and the progress to date on the solution of these problems. We discuss in Section 1.3 the scope of the thesis, highlight the key contributions, and provide a chapter-by-chapter outline of the contents of this thesis.

1.1 The Nature of the Problems Discussed in This Thesis

The advent of power interchange between interconnected power systems brings about a key need to understand the nature of the economic and physical aspects of each of the interconnected power systems, as well as the coordination scheme that governs the power interchange among interconnected power systems. The power system operations and wholesale purchase/sale of electricity in each individual power system are administered by an independent entity. We refer to this independent entity by the generic name *independent grid operator* (IGO) to encompass various organizations such as independent system operator (ISO), transmission system operator (TSO), regional transmission organization (RTO) and independent transmission provider (ITP).

Power systems may be connected to each other via-tie lines, which allow the flow of power among interconnected power systems. The interconnectivity of the power systems engenders various opportunities for the IGOs of these systems, including the coordination of their power system operations and wholesale purchase/sale of electricity. An obvious goal for such a coordination is to meet the power demand with a lower total payment. After the resources in a system are used to clear the demand in that system, additional available generation can be exported to other interconnected systems via the utilization of the tie lines, as long as it is cheaper than the generation in the importing system. The IGOs of the interconnected systems need to coordinate with each other to perform such an interchange. Since IGOs cannot take financial positions in electricity markets, they cannot buy/sell power from/to each other. Hence, they need to harness market-based coordination schemes for the determination of interchange evaluation. One such scheme is coordinated transaction scheduling (CTS), which constitutes the focus of this thesis [1]. CTS has been approved by the Federal Energy Regulatory Commission ($FERC$) for the New York Independent System Operator ($NYISO$) and the Independent System Operator-New England (ISO-NE) in 2012, and has been in place for the determination of their interchange evaluation since December 15, 2015 [2], [3].

The execution of a coordination scheme requires the IGOs of the interconnected systems to consider the power system operations and the wholesale purchase/sale of electricity of each interconnected system, as well as the procedural steps of the coordination scheme. However, an IGO is required to maintain the confidentiality of certain data/information on its system. Such a requirement limits the data/information that is made available for the execution a coordination scheme. Regardless of the particular choice of coordination scheme, the IGOs of the interconnected power systems must agree on which data/information to share, and must maintain the confidentiality of the unshared information as the private information of each IGO.

The execution of any coordination scheme involves many sources of uncertainty [4]. A coordination scheme is executed for a *coordination period* based on certain data/information that is collected at a time instant before the beginning of that coordination period. The collected data/information for the execution of a coordination scheme may include the demand, the generation of the resources, power flow data, status of the tie-lines etc. However, the collected data/information may change during the coordination period. For instance, the generators that were expected to be available may not be available, the actual demand may be different than the forecasted demand, or tie-lines can fail. Hence, we are interested in decreasing the coordination period durations so as to capture the actual data/information better.

Such are the topics that we explicitly consider in this thesis. Before we delve into these topics in detail, we first review the state of the art on the area in the next section.

1.2 Review of the State of the Art

In this section, we briefly review the coordination schemes that are deployed in the actual power system operations, as well as the literature on coordination schemes. The capability of transaction of power between interconnected systems may predate the utilization of coordination schemes for interchange. A scheme that did not involve the coordination of the IGOs and was frequently used was that an external market participant that wished to transact power between two interconnected power systems would separately submit an offer to one IGO and a bid to the other IGO. The external market participant would submit an offer/bid to the IGO from/to which it wished to transact power. The submitted offer and bid were cleared separately in the markets of the IGOs. If the offer price of the external market participant is higher than the price of electricity in the system from which it wishes to transact power, and the bid price of the external market participant is lower than the price of electricity in the system to which it wishes to transact power, then the transaction of power would clear. Such a scheme clearly lacked the coordination of the two IGOs, and resulted in power flows from the system where the price of electricity was higher to the system where the price of electricity is lower—which is contrary to the goal of the utilization of interchange [1].

In December 15, 2015, *ISO-NE* and *NYISO* started to use a coordinated scheme called *coordinated transaction scheduling (CTS)*—which also constitutes the focus of this thesis [3]. *CTS* involves an offer type called an *interface offer* which is submitted simultaneously to both *IGO*s by an external market participant that wishes to transact power across the interface [1]. The two *IGO*s use the submitted interface offers, and share some of their data/information regarding submitted offers and demands to determine the interchange evaluation. However, each *IGO* must also maintain the confidentiality of data/information on its system, which limits the available data/information for the execution of *CTS*. The sharing of data/information between the interconnected *IGO*s is considered in detail throughout the thesis.

Another scheme that is discussed in the industry is *tie-line optimization (TLO)* [1]. *TLO* assumes the hypothetical aggregation of two interconnected systems, and that the hypothetically aggregated *super-system* is operated by an hypothetical super-*IGO* that overtakes the functions of both *IGO*s and so has knowledge of the offers and the demands of the two *IGOs*. Hence, *TLO* does not recognize either the identities or the independence of the two interconnected *IGOs*. Therefore, the determination of what used to be referred to as interchange via *TLO* is the same as the determination of power flow on any transmission line of the *super-system*.

Interchange evaluation has been discussed in the literature since the 1980s, mainly in the form of a multi-area coordination problem[4], [5]. The solution to such problems generally employs methods that decompose the joint optimal power flow problem, and attain the optimal power flow by iteratively exchanging information. The purpose of such solution methods is to drive the dispatch in each area toward the optimal solution of the joint optimal power flow problems, hence attaining overall efficiency. As briefly reviewed in [5], some of the methods involving primal decomposition methods are provided in [6] and [7], and examples of solution methods involving dual decomposition methods can be found in [8] and [9]. However, such approaches do not conform to the electricity market structure in North America, because they require ISOs to take financial positions in the markets.

1.3 Scope and Contributions of This Thesis

In this thesis, we study coordinated transaction scheduling (CTS)—a coordination scheme for the determination of interchange evaluation. We are interested in the analysis and the quantification of the performance of CTS. In addition, we would like to have a consistent basis that allows the side-by-side comparison of CTS with any other coordination scheme. The discussion in Section 1.1 indicated that, for the analysis and the evaluation of the performance of a coordination scheme, it is required to explicitly represent the physical aspects, economic aspects, steps of the coordination procedure, and the interactions among them. In order to both aid the analysis and the quantification of the performance of a coordination scheme, and to have a consistent basis that allows the comparison of any two coordination schemes, we construct a two-layered framework. The framework is general, it can be used for the evaluation and the performance metric evaluation of any coordination scheme, and can accommodate any choice of representation. In addition, the framework is comprehensive as it accommodates the representation of all required aspects involving the analysis and the performance metric evaluation of a coordination scheme.

For the analysis and the quantification of the performance of CTS, we tailor this framework for CTS and develop appropriate models for the required representation. As discussed in Section 1.1, in order for the IGOs of the interconnected systems to execute a coordination scheme, they need to share certain data/information and must maintain the confidentiality of the unshared information as their private information. We represent the flow of data/information between the IGOs by the interactions between the respective models of the framework. The respective models of the IGOs accommodated in the framework interact with each other according to the sharing specifications of data/information. CTS consists of procedural steps that utilize these data/information, and the steps of the CTS must be executed sequentially by the interconnected IGOs in a coordinated way. We use the framework for the provision of an analytical underpinning of CTS, and the formulation of the required tasks in the procedural steps of CTS. We illustrate the execution of CTS step-by-step using two sample data sets. We conduct various sensitivity analyses, and study the dependence of the interchange amount and total payments on certain data/information regarding the physical and economic aspects of the interconnected power systems.

We consider the elapsed time durations of the procedural steps of CTS, and identify the binding constraints for each procedural step so as to work toward shorter coordination periods. The discussion in Section 1.1 pointed out the importance of coordination period duration for interchange evaluation. We study the dependence of the demand forecast performance on the coordination period duration. In order to observe the dependence of interchange on the coordination period duration, we use real-time market prices to simulate the interchange for different coordination periods. Such a simulation allows us to observe the impact of shorter coordination period duration on interchange. We also study TLO, and use the results obtained via TLO as a benchmark to compare the results obtained via CTS.

The thesis contains five additional chapters. In Chapter 2, we construct a framework for interchange evaluation methodologies. This framework consists of two interconnected layers: physical layer and economic layer. We describe the layers of the framework, as well as possible interactions of the models that get accommodated in the layers of the framework. In Chapter 3, we study the procedural steps of CTS. We tailor the framework constructed in Chapter 2 for CTS, develop the power system and market models that get accommodated in the respective layers of the framework, and formulate the procedural steps of CTS. We discuss the rationale of certain tasks in the procedural steps of CTS, and provide insights into how certain tasks influence the total payments and the interchange amount. In Chapter 4, we provide illustrative examples for the execution of CTS, and provide a step-by-step explanation. We conduct various sensitivity analyses, and study the dependence of the interchange amount and total payment on certain data/information regarding the physical and economic aspects of the interconnected power systems. In Chapter 5, we assess the time duration of each procedural step of CTS. We identify the limiting constraints of the time duration of each step so as to assess the feasibility to undertake CTS with shorter coordination periods. We perform simulations for different coordination periods, and discuss the results of these simulations and provide important insights into the impacts of coordination period durations. In Chapter 6, we provide our concluding remarks together with a summary of this thesis, and discuss directions for future research to extend the results in this thesis. The thesis has two appendices. In Appendix A, we provide the notation used in this thesis. We dedicate Appendix B to TLO. We develop appropriate power system and market models, and present the formulation of the procedural steps of TLO.

CHAPTER 2

FRAMEWORK FOR INTERCHANGE EVALUATION METHODOLOGIES

The review in Chapter 1 makes it clear that the analysis and the quantification of the performance of the diverse interchange schemes—with their salient characteristics and specific structures, and their associated information needs—are challenging problems. From now on, for the purposes of brevity, we refer to the interchange evaluation scheme, its characteristics and structure, and its associated information needs by the all-inclusive term *interchange evaluation methodology.* The interchange evaluation decision is fraught with many sources of uncertainty, including the demand forecast, the availability of conventional resources, and the time-varying, uncertain, and intermittent nature of renewable energy resources. Moreover, for each side of the interchange, there is price uncertainty with respect to the other side. The interchange evaluation methodologies require that data be shared between the two interconnected IGOs on a symmetric basis, and that they jointly participate in the performance of the coordination scheme. However, each IGO wants to maintain the confidentiality of the data/information of its system, and prefers to share as little data/information as possible, which limits the information that is made available for the decision of interchange evaluation. The discussion of TLO in Chapter 1 makes clear that TLO is an ideal case that assumes the hypothetical aggregation of the two interconnected systems. If this ideal were possible, there would no longer be the problem of interchange, because the hypothetical aggregation would result in a single system. Unfortunately, such a hypothetical aggregation is not realistic under current conditions, as no IGO would be willing to give the control of its system to a higher authority.

The analysis and the quantification of the performance of any interchange evaluation methodology require the explicit representation of the physical aspects, the economic aspects, the steps of the coordination scheme, and all the interactions among them. The physical aspects include the power system components of each IGO that are deployed for interchange, as well as the associated power flows. The economic aspects include the determination of the costs and benefits associated with the wholesale purchase/sale of electricity performed in each system, with the particular coordination scheme explicitly taken into account. Clearly, the determination of the costs and benefits may or may not be market-based. While each IGO has the data/information on its physical aspects and its economic aspects, the interchange requires that some subset of data/information is shared with the neighboring IGO.

The analysis and the performance evaluation for each methodology can be greatly aided by the construction of a framework that accommodates the required representation of the physical aspects, the economic aspects, the steps of the coordination scheme, and all the interactions among them. However, in light of the diversity of the extant methodologies, the fact that a framework is suitable for a particular interchange evaluation methodology does not imply its suitability for any other interchange evaluation methodology. Hence, we cannot use any framework that is suitable for a particular interchange evaluation methodology as a basis that allows the comparison of any two interchange evaluation methodologies in a consistent way.

As such, it makes sense to construct a common framework that can incorporate the salient characteristics of each methodology in terms of the appropriate models and tools needed for the analysis and the performance evaluation. Such a common framework allows the side-by-side comparison of any two interchange evaluation methodologies and may be used to accommodate each of the methodologies in Chapter 1 and proposed future methodologies.

The two requirements of the framework are generality and comprehensiveness. The framework must be sufficiently general to include all existing and possibly proposed interchange schemes. As existing and possibly proposed schemes may or may not be market-based, the framework must allow the representation of both market-based and non-market-based schemes as appropriate to the particular situation. The framework must also be comprehensive and accommodate the representation of the power system components and the associated power flows, as well as the economic representation and the coordination scheme of the interchange. In addition, the framework must also be able to accommodate all data/information flows required by the interchange evaluation methodology.

We devote this chapter to the development of such a framework for interchange evaluation methodologies. We explain the design and construction of the framework in Section 2.1. In Section 2.2 we describe the physical layer, and in Section 2.3 we discuss the economic layer of the framework. We summarize the chapter and state our concluding remarks in Section 2.4.

2.1 Design and Construction of the Framework

In this thesis, we focus on the intra-hourly subperiod $[k]\big|_h$, and consider $[k]\big|_h$ as the *coordination period* for interchange evaluation. For a given day, we express all time elements in minutes (*min*) and a time element τ denotes the cumulative minutes elapsed from the start of the day, at 0 *min*. The term h is the index for the hourly periods of a day such that $h = 1, 2, ..., 24$. We define the hourly period as $\mathcal{T}_h \triangleq \{\tau : (h-1)(60) < \tau \leq (h)(60)\}$.

We consider K intra-hourly subperiods of equal duration for each hour h, and introduce the index k for the intra-hourly subperiods such that $k = 1, 2, ..., K$. We denote the duration of each intra-hourly subperiod by ζ. We define the intra-hourly subperiod as $\mathcal{T}_{[k]\big|_h} \triangleq \{\tau : (h-1)(60) + (k-1)(\zeta) < \tau \leq (h-1)(60) + (k)(\zeta)\}$. The time frame of our analysis is illustrated in Figure 2.1.

We construct the framework for interchange evaluation methodologies for two in-

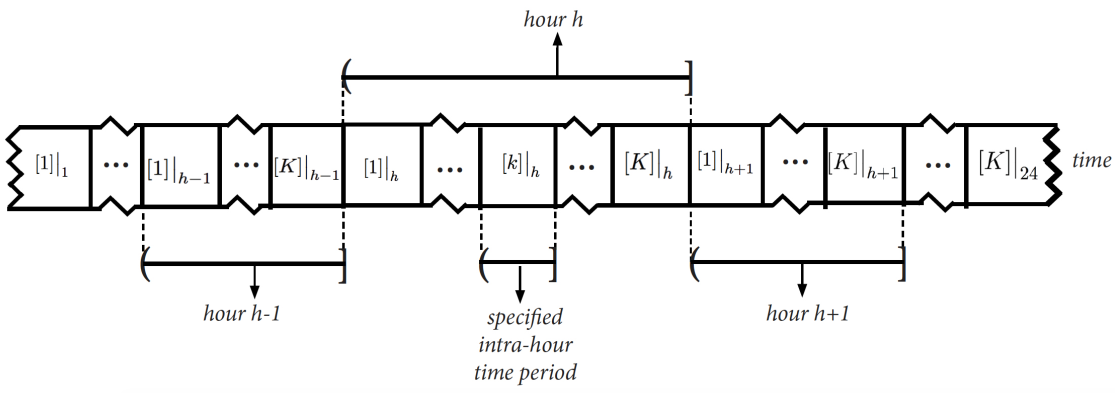

Figure 2.1: Time scale of the framework

terconnected IGOs: IGO^A and IGO^B. In order to meet the requirement of the explicit representation of the physical aspects and the economic aspects, the framework is designed to consist of two interconnected layers: the physical layer and the economic layer. The physical layer accommodates the representation of the power system components and the associated power flows of the two systems that are deployed by the particular interchange methodology. The representation of the power system components and the associated power flows is done via the development of appropriate models. The choice of the particular model depends on the particular interchange evaluation methodology, as well as the level of detail required by the specific application. The physical layer of the framework can accommodate any choice of model for the representation of the physical assets and the associated power flows of the two systems.

The economic layer accommodates the economic representation of the two interconnected systems and the coordination scheme. The economic representation of the systems and the particular choice of coordination scheme is done via the deployment of appropriate models. Although the economic representation of the systems are mostly market-based in North America, in order to render the framework general, we do not specify the economic models of the systems. The economic layer is able to accommodate any choice of model for the economic representation of the systems and the coordination scheme.

The models in the physical layer and the economic layer interact with each other via data/information flows. The flow of data/information can be between the models in the same layer, as well as between the models in different layers. The framework is able to accommodate any flow of data/information between any models in the two layers. The data/information can take many forms, including power flow, sensor measurements, and economic information. The framework is able to distinguish the different forms of data/information that is exchanged between the models. The share of data/information between the two interconnected *IGO*s is a critical matter. Each *IGO* is the single entity responsible for the administration of its system, and it is in the interest of each IGO to maintain the confidentiality of the data/information of its system. However, the coordination scheme may require the sharing of certain data/information. The framework is able to distinguish the sharing specifications of all data/information, and is able to accommodate the sharing of data/information between only the respective models. The general structure of the framework is provided in Figure 2.2.

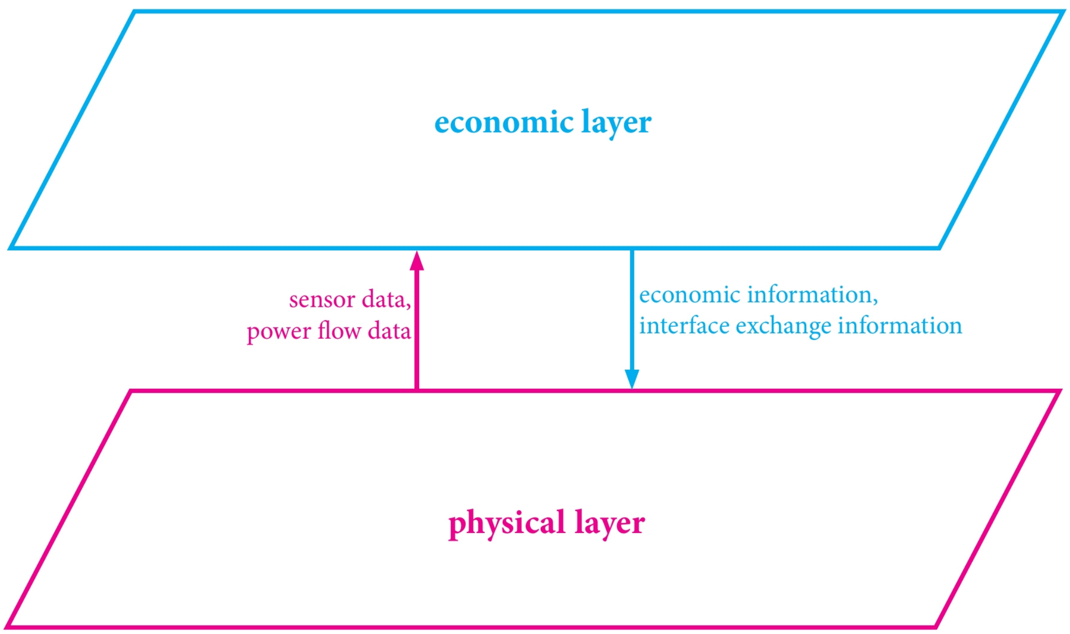

Figure 2.2: General structure of the framework

2.2 The Physical Layer

We designed the proposed framework to consist of the physical layer and the economic layer. The physical layer accommodates the representation of the power system components and the associated power flows of the two systems that are deployed for the decision of interchange evaluation. The representation of the power system components and the associated power flows of the interchange is done via the deployment of appropriate models. The general structure of the physical layer is provided in Figure 2.3.

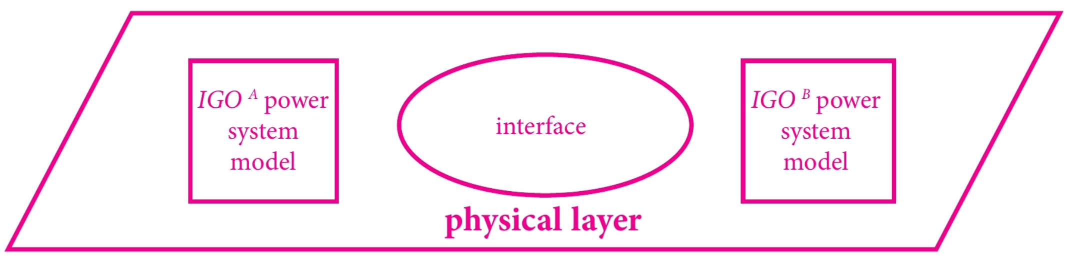

Figure 2.3: Physical layer

The physical layer accommodates the IGO^A and IGO^B power system models. Considering the actual power system components, each IGO operates its transmission network, which contains a certain number of busses, internal transmission lines, and transformers. The generators that are connected to a bus of the transmission network inject power, and the loads in each bus withdraw power. The power system model of each IGO contains an abstract representation of these power system components and the associated power flows, and regardless of the particular choice of model, the physical layer can accommodate any choice for the IGO^A and IGO^B power system models.

The physical power systems of IGO^A and IGO^B are connected via tie line(s). A tie-line is a physical transmission line that connects a bus in one IGO to another bus in the other IGO. There may be one or more physical tie-lines between the two interconnected IGOs, and the power flows between the two interconnected IGOs through the tie-line(s). For the purposes of interchange evaluation decision, the exchange of power between the two interconnected IGOs is modeled to be on the *interface* between the two IGOs. The physical layer also accommodates the interface, regardless of the particular choice of model.

The choice of the particular models that are accommodated in the physical layer is commensurate with the needs of the particular interchange evaluation methodology and the application, and the physical layer is able to accommodate any choice of model for the IGO^A power system, the IGO^B power system, and the interface.

2.3 The Economic Layer

The other layer in the proposed framework is the economic layer. The economic layer accommodates the economic representation of the two systems and the coordination scheme. The economic representation of the systems and the coordination scheme is done via the deployment of appropriate models. The general structure of the economic layer is provided in Figure 2.4.

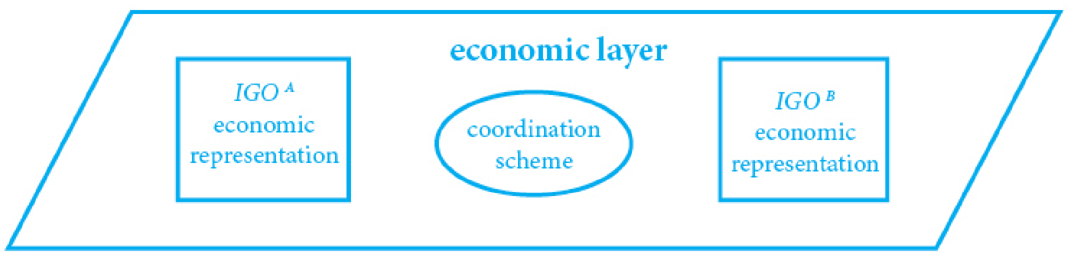

Figure 2.4: Economic layer

Each system has a certain economic scheme by which the wholesale purchase/sale of electricity is performed. In the systems in North America, the wholesale purchase/sale of electricity is performed usually via an electricity market that that has a uniform-price double-sided auction mechanism. However, different schemes may also be used by an *IGO* to perform the wholesale purchase/sale of electricity. The economic scheme by which the wholesale purchase/sale of electricity is performed in each IGO is represented via the economic model of that IGO. Regardless of the particular choice of model, the economic layer of the proposed framework accommodates the IGO^A economic system model and the IGO^B economic system model.

The coordination scheme is the specific mechanism that is jointly deployed for the decision of interchange evaluation by the two interconnected IGOs. The economic layer of the proposed framework also accommodates any choice of coordination scheme.

The framework allows the interaction of any models in any two layers of the framework. The interactions may be in the form of power flow data, sensor measurements and economic information. Depending on the specific interchange methodology and the application for which the framework is used, the represented flows of data/information between the models may differ. The power system model of a system in the physical layer may pass sensor measurements and power flow data to the economic model of that system in the economic layer. The economic models of the systems may exchange certain economic information with the coordination scheme. The coordination scheme in the economic layer may pass the evaluated interchange flow amount to the interface in the physical layer. The framework is able to accommodate any of these flows of data/information, and distinguish the form of the passed data/information.

Given that each *IGO* wants to maintain the confidentiality of its data/information and wants to share as little data/information as possible, the framework is designed so as to consider the sharing specifications of data/information. Hence, the framework enables the sharing of data/information to be between only the specified models. The detailed structure of the framework is provided in Figure 2.5.

2.4 Concluding Remarks and Summary

In this chapter, we have described a framework for the analysis and the quantification of the performance of interchange evaluation methodologies. In order to meet the requirements for the explicit representation of the physical aspects, the economic aspects, the interactions between the economic aspects, and the steps of the coordination scheme, we have constructed the framework to consist of two interconnected layers: the physical layer and the economic layer. We have designed the framework so as to render it general and comprehensive for the analysis and the performance evaluation of interchange evaluation methodologies. The framework is general, as it can be used to do the analysis and performance evaluation of any interchange evaluation methodology, and can accommodate any physical model, economic model, and coordination scheme in its respective layers. The generality of the framework enables it to serve as a consistent basis for the side-by-side comparison of interchange evaluation methodologies. The framework is comprehensive, as it accommodates all the models and tools required for the decision of interchange evaluation, as well as their interactions with each other, in a unified construct. In the following chapters, we will apply the framework introduced in this chapter to the analysis and the quantification of the performance of coordinated transaction scheduling (CTS).

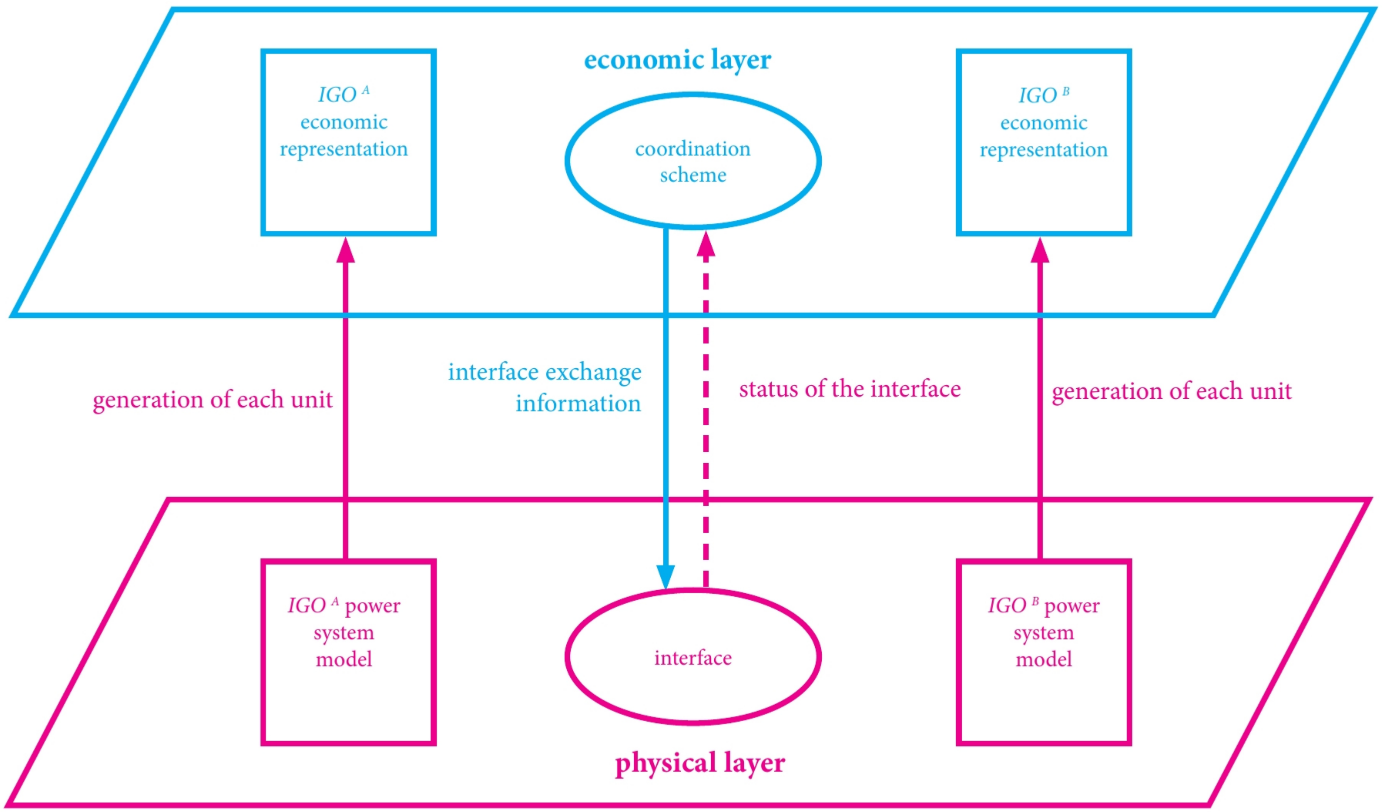

Figure 2.5: Detailed structure of the framework

CHAPTER 3

PROCEDURAL STEPS OF COORDINATED TRANSACTION SCHEDULING

In this chapter, we discuss coordinated transaction scheduling (CTS), which is a scheme that is jointly deployed by two interconnected IGOs to determine their interchange evaluation.[1] We have constructed the framework in Chapter 2 so as to make it general and comprehensive, which enabled the framework to be used for the analysis and the quantification of the performance of any interchange evaluation methodology. In this chapter, we tailor the framework constructed in Chapter 2 for CTS, and present the models that are accommodated in the physical and economic layer of the framework, as well as the interactions between the models. Using the

[1] Our description of CTS in this chapter is based on [1], which has been deployed by the Independent System Operator-New England (ISO-NE) and New York Independent System Operator ($NYISO$) since December 15, 2015.

CTS framework, we present the procedural steps of CTS, and describe the execution of each step considering the required data/information exchange between the models. Throughout the chapter, we adopt the time convention that is explained in Section 2.1, and focus on the coordination period $[k]\big|_h$ to discuss the execution of CTS. In Section 3.1, we tailor the framework described in Chapter 2 to develop the CTS framework. We devote Sections 3.2–3.5 to discuss of the steps of CTS. Finally, we provide the concluding remarks and the summary of the chapter in Section 3.6.

3.1 The CTS Framework

In the previous chapter, we have presented a general and comprehensive framework that could be used for the analysis and the performance evaluation of any interchange evaluation methodology. In this section, we tailor the framework constructed in Chapter 2 for CTS and provide the specific models that are accommodated in the framework. While constructing the framework in Chapter 2, we denoted the two interconnected IGOs as IGO^A as IGO^B. In the CTS framework—for reasons that will become clear in the rest of the chapter—we refer to IGO^A as IGO^M and IGO^B as IGO^F.

In order to accommodate the explicit representations of the physical aspects, economic aspects, the interactions between the physical and economic aspects, and the coordination scheme, we have constructed the framework to consist of two interconnected layers: the physical layer and the economic layer. The physical layer of the framework accommodates the IGO^M power system model, IGO^F power system model, and the interface.

Each IGO operates its transmission network, which contains a certain number of busses, internal transmission lines, and transformers. The generators that are connected to a bus of the transmission network inject power, and the loads in each bus withdraw power. For the purposes of interchange evaluation, we model the power system of each IGO such that each power system contains a single bus. We denote the bus of IGO^M by † and the bus of IGO^F by ‡. In addition to a single bus, the power system model of each IGO contains the information on the generators in that system.

The two power systems are physically connected via tie-line(s). There may be one or more physical tie-lines between the two interconnected IGOs, and the power flows between the two interconnected IGOs through these tie-line(s). For the purposes of interchange evaluation, we model the power exchange between the two interconnected *IGO*s to be on the *interface* between † and ‡. Hence, IGO^F proxy bus for IGO^M is †, and IGO^M proxy bus for IGO^F is ‡. We denote the interface exchange amount by f_ℓ, and use the convention that if the power exchange is from ‡ to †, then $f_\ell > 0$, and if the power exchange is from † to ‡, then $f_\ell < 0$. We assume that the interface is able to accommodate any interface exchange amount.

The other layer in the framework constructed in Chapter 2 is the economic layer, which accommodates all the economic aspects of interchange evaluation and the coordination scheme. We introduce the IGO^M (IGO^F) internal market model, which is the mechanism by which IGO^M (IGO^F) performs the wholesale purchase/sale of electricity of its system. We consider that IGO^M and IGO^F market models have a uniform-price single-sided auction mechanism. The internal market players of each *IGO* sell energy directly to the *IGO* by submitting sealed internal offers. Each such internal offer i (j)—where $i = 1, 2, ..., I$ ($j = 1, 2, ..., J$)—that is submitted to the bus of IGO^M (IGO^F) is represented by the couplet $\beta_i^\dagger = \{p_i^\dagger, \mu_i^\dagger\}$ ($\beta_j^\ddagger = \{p_j^\ddagger, \mu_j^\ddagger\}$), where $p_i^\dagger$ ($p_j^\ddagger$) is the offered active power in MW and $\mu_i^\dagger$ ($\mu_j^\ddagger$) $\$/MWh$ is the offer price. We represent the set of all internal offers submitted at † (‡) by $\{\beta^\dagger\}$ ($\{\beta^\ddagger\}$).

In our analysis of the internal markets for the coordination period, we consider the intra-hour demand in the market model of each IGO to be price-insensitive. We refer to the intra-hour demand of each IGO as the residual demand that is not cleared in the day-ahead market and that needs to be cleared during the coordination period $[k]\big|_h$ either by the IGO's internal offers or by importing power from the neighboring IGO. We represent the intra-hour demand at the bus † (‡) with $d^{\dagger}$ ($d^{\ddagger}$). The intra-hour demand considered in this chapter is essentially a forecasted demand, which we assume to remain constant during the coordination period. In practice, the demand may differ from the forecasted value and change within the coordination period. We study the dependence of the CTS results on the duration of the coordination period in Chapter 5.

The economic layer also accommodates the CTS model, in which the procedural steps of CTS are executed. CTS includes an offer form called *interface offer*[2] which is an offer submitted to transact power from one bus to the other on the interface. An interface offer c (n) that expresses the willingness to transact $p_c^{\ddagger\rightarrow\dagger}$ ($p_n^{\dagger\rightarrow\ddagger}$) MW from ‡ (†) to † (‡) under the condition that $\chi_{\dagger} - \chi_{\ddagger} \geq \mu_c^{\ddagger\rightarrow\dagger}$ ($\chi_{\ddagger} - \chi_{\dagger} \geq \mu_n^{\dagger\rightarrow\ddagger}$) is expressed as the couplet $\beta_c^{\ddagger\rightarrow\dagger} = \{p_c^{\ddagger\rightarrow\dagger}, \mu_c^{\ddagger\rightarrow\dagger}\}$ ($\beta_n^{\dagger\rightarrow\ddagger} = \{p_n^{\dagger\rightarrow\ddagger}, \mu_n^{\dagger\rightarrow\ddagger}\}$). Hence, the interface offer corresponds to the offer of $p_c^{\ddagger\rightarrow\dagger}$ ($p_n^{\dagger\rightarrow\ddagger}$) MW of withdrawal at

[2]In [1], the term *interface bid* is used instead of *interface offer*.

$\ddagger$ ($\dagger$) and $p_c^{\ddagger\rightarrow\dagger}$ ($p_n^{\dagger\rightarrow\ddagger}$) of injection at $\dagger$ ($\ddagger$) as long as the marginal price at $\dagger$ ($\ddagger$) exceeds that of $\ddagger$ ($\dagger$) at least by $\mu_c^{\ddagger\rightarrow\dagger}$ ($\mu_n^{\dagger\rightarrow\ddagger}$). A distinguishing feature of an interface offer from an internal offer is that an interface offer is submitted on the basis of marginal price difference between $\dagger$ and $\ddagger$, and is indifferent to the actual marginal price values at $\dagger$ and $\ddagger$.

The models described above get accommodated in their respective layers of the framework. Specifically, IGO^M power system model, IGO^F power system model, and the interface get accommodated in the physical layer, and the IGO^M internal market model, IGO^F internal market model, and the CTS model get accommodated in the economic layer. Each IGO needs to collect the required data and prepare the necessary information on its system through the appropriate models prior to the execution of CTS. The models in the framework interact with each other through the flow of these data/information. Whether the model of an IGO shares its data/information with the model of a neighboring IGO is a critical matter, because each IGO wants to maintain the confidentiality of its data/information. Hence, we decide on the sharing of the data/information so as to allow the CTS to run with minimum data/information exchange between the models of the two interconnected IGOs.

The status of the generators in each power system model in the physical layer is passed to its respective internal market model in the economic layer, and then is passed to the CTS model in the economic layer. The status of the interface is passed from the physical layer to the CTS model in the economic layer. Each IGO collects its internal offers from the internal market participants, and obtains the intra-hour demand in its internal market model. The internal offers and the intra-hour demand of each IGO are passed from its respective internal market model in the economic layer to the CTS model in the economic layer. The interface offers are also directly submitted to the CTS model in the economic layer. CTS is executed in the economic layer of the framework using the data/information that is passed to the CTS model. In the following sections, we explain the procedural steps of CTS.

3.2 Step 1: Determination of the Interface Exchange Direction

The initial step of CTS is to determine the direction of the interface exchange. This is a critical step because CTS ensures that the power exchange on the interface is from the bus with the lower marginal price to that with the higher marginal price.[3]

[3]The prices that we refer to in this chapter are all *ex-ante* prices that are determined based on the forecasted intra-hour demands and the submitted offers. The *ex-post* prices may different as

In this step, we assume that each IGO meets its forecasted intra-hour demand without resorting to the interface exchange. Such an assumption enables us to obtain what the marginal price of electricity at each bus would have been, had there been no utilization of the interface exchange for the intra-hour market. Based on these marginal prices that are obtained without resorting to the interface exchange, we designate the interface exchange direction so that it is from the bus with the lower marginal price to that with the higher marginal price.

For the determination of the interface exchange direction, the CTS model solves an ED problem for each IGO using the submitted internal offers and its intra-hour demand. In order to formulate the ED problem, we introduce the variable $\acute{a}_i^{\dagger}$ ($\acute{a}_j^{\ddagger}$), which represents the cleared real power in MW from the i^{th} (j^{th}) submitted internal offer. Hence, $\acute{a}_i^{\dagger}$, $i = 1, 2, ...I$ ($\acute{a}_j^{\ddagger}$, $j = 1, 2, ...J$) serve as the *optimization variables* in the ED problem formulation of Step 1.

The ED problem at $\dagger$ is formulated as:

the actual intra-hour demand may differ from the forecasted intra-hour demand. Although CTS ensures that the interface exchange direction is from the bus with the lower *ex-ante* marginal price to that with the higher *ex-ante* marginal price, it cannot ensure the same for *ex-post* marginal prices. We discuss this issue in detail in Chapter 5.

$$
\begin{aligned}
&\min_{\acute{a}_i^{\dagger}} \quad \sum_{i=1}^{I} (\acute{a}_i^{\dagger})\,(\mu_i^{\dagger}) \\
&s.t. \sum_{i=1}^{I} \acute{a}_i \,-\, \bar{d}^{\dagger} \,=\, 0 \qquad \longleftrightarrow \; \acute{\chi}^{\dagger} \\
&\acute{a}_i \,-\, p_i \leq 0 \quad i \,=\, 1, 2, ...I
\end{aligned}
\tag{3.1}
$$

The optimal decision variables $\acute{a}_i^{\dagger *}$ $i \,=\, 1, 2, ...I$ indicate the MW of power cleared from the internal offers $i \,=\, 1, 2, ...I$ by solving (3.1). The exact same problem is solved for IGO^F and the optimal decision variables $\acute{a}_j^{\ddagger *}$ $j \,=\, 1, 2, ...J$ are obtained for IGO^F.

We denote the dual variable associated with the equality constraint at the optimum for IGO^M (IGO^F) as $\acute{\chi}^{\dagger\,*}$ ($\acute{\chi}^{\ddagger\,*}$). $\acute{\chi}^{\dagger\,*}$ and $\acute{\chi}^{\ddagger\,*}$ imply the marginal price to serve an additional MW of demand at † and ‡, respectively. Based on $\acute{\chi}^{\dagger\,*}$ and $\acute{\chi}^{\ddagger\,*}$, the CTS model determines which interface offers are incorporated to the following steps of CTS, and which interface offers are not considered. If $\acute{\chi}^{\dagger\,*} < \acute{\chi}^{\ddagger\,*}$, then the interface offers submitted to transact power from † to ‡ are incorporated into the following steps of CTS, and the interface offers submitted to transact power from ‡ to † are not considered in the subsequent steps. On the other hand, if $\acute{\chi}^{\ddagger\,*} < \acute{\chi}^{\dagger\,*}$, then the interface offers submitted to transact power from ‡ to † are incorporated into the following steps of CTS, and the interface offers submitted to transact power from † to ‡ are not considered in the subsequent steps.

The incorporation of the interface offers is critical in CTS because the interface offers can be submitted to transact power in either direction. Hence, the interface offers can be submitted to transact power from the bus with the higher marginal price to the bus with the lower marginal price, which is contrary to the aim of the interface and is economically counter-intuitive. The motivation behind the incorporation of only the interface offers that are submitted in the direction from the bus with the lower marginal price to that with the higher marginal price is to ensure that the interface exchange is economically intuitive and that such economically counter-intuitive interface offers are not considered. Once Step 1 is completed, the two IGOs know the direction of the interface exchange, which is aligned with the intuitive notion of exchange from the bus of the IGO with the lower marginal price to the bus of the IGO with the higher marginal price.

3.3 Step 2: Modification of the Internal Offers at the Bus of IGO^M

In this step, the interface offers are matched against the internal offers at † so as to determine the possible modifications of the internal offers at †. The internal offers at the bus † of IGO^M are modified by the interface offers, and the internal offers at the bus ‡ of IGO^F remain fixed, hence the renaming of IGO^A as IGO^M and IGO^B as IGO^F in this chapter.

We first explain the matching and the modification tasks for the case that interface exchange is from ‡ to †. The tasks consist of the following substeps:

Substep $2.\,a\,(i)$: Arrange the interface offers from ‡ to † $\{\beta^{\ddagger\to\dagger}\}$ in the order of increasing price differences to obtain $\mu_1^{\ddagger\to\dagger} \leq \mu_2^{\ddagger\to\dagger} \leq ... \leq \mu_C^{\ddagger\to\dagger}$. Arrange the internal offers $\{\beta^{\dagger}\}$ in the order of decreasing offer prices to obtain $\mu_1^{\dagger} \geq \mu_2^{\dagger} \geq ... \geq \mu_I^{\dagger}$.

Substep $2.\,a\,(ii)$: Set $i = 1$ and $c = 1$. If $C < I$; go to **Substep** $2.\,a\,(iii)$; otherwise, go to **Substep** $2.\,a\,(v)$.

Substep $2.\,a\,(iii)$: Modify the internal offer $\beta_i^{\dagger}$ to create $\tilde{\beta}_i^{\dagger} = \{p_c^{\ddagger\to\dagger}, \mu_i^{\dagger} - \mu_c^{\ddagger\to\dagger}\}$ and $\tilde{\beta}_{I+i}^{\dagger} = \{p_i^{\dagger} - p_c^{\ddagger\to\dagger}, \mu_i^{\dagger}\}$. Set $i = i + 1$ and $c = c + 1$.

Substep $2.\,a\,(iv)$: If $c = C$, terminate; otherwise, return to **Substep** $2.\,a\,(iii)$.

Substep $2.\,a\,(v)$: Modify the internal offer $\beta_i^{\dagger}$ to create $\tilde{\beta}_i^{\dagger} = \{p_c^{\ddagger\rightarrow\dagger}, \mu_i^{\dagger} - \mu_c\}$ and $\tilde{\beta}_{I+i}^{\dagger} = \{p_i^{\dagger} - p_c^{\ddagger\rightarrow\dagger}, \mu_i^{\dagger}\}$. Set $i = i + 1$ and $c = c + 1$.

Substep $2.\,a\,(vi)$:If $i = I$, terminate; otherwise, return to **Substep** $2.\,a\,(v)$.

Substep $2.\,a\,(vii)$: Arrange the internal offers at † after modification $\{\tilde{\beta}^{\dagger}\}$ in the order of decreasing offer prices to obtain $\tilde{\mu}_1^{\dagger} \geq \tilde{\mu}_2^{\dagger} \geq ... \geq \tilde{\mu}_{\tilde{I}}^{\dagger}$.

The number of internal offers at † after the modification is denoted as $\tilde{I}$. If $I \geq C$, then C of the offers are modified to create $2C$ offers, and $I - C$ offers remain unchanged; therefore, $\tilde{I} = I + C$. If $I < C$, then the I offers are modified to create $2I$ modified offers, and each initial offer is modified; therefore, $\tilde{I} = 2I$.

For the case that the interface exchange is from † to ‡, the matching and the modification tasks consist of the following substeps:

Substep $2.\,b\,(i)$: Arrange the interface offers from † to ‡ $\{\beta^{\dagger\rightarrow\ddagger}\}$ in the order of increasing price differences to obtain $\mu_1^{\dagger\rightarrow\ddagger} \leq \mu_2^{\dagger\rightarrow\ddagger} \leq ... \leq \mu_N^{\dagger\rightarrow\ddagger}$. Arrange the internal offers $\{\beta^{\dagger}\}$ in the order of decreasing offer prices to obtain $\mu_1^{\dagger} \geq \mu_2^{\dagger} \geq ... \geq \mu_I^{\dagger}$.

Substep $2.\,b\,(ii)$: Set $i = 1$ and $n = 1$. If $N < I$; go to **Substep** $2.\,b\,(iii)$; otherwise; go to **Substep** $2\,.b\,(v)$.

Substep $2.\,b\,(iii)$: Modify the internal offer $\beta_i^{\dagger}$ to create $\tilde{\beta}_i^{\dagger} = \{p_n^{\dagger\to\ddagger}, \mu_i^{\dagger} + \mu_n^{\dagger\to\ddagger}\}$ and $\tilde{\beta}_{I+i}^{\dagger} = \{p_i^{\dagger} - p_n^{\dagger\to\ddagger}, \mu_i^{\dagger}\}$. Set $i = i+1$ and $n = n+1$.

Substep $2.\,b\,(iv)$: If $n = N$, terminate; otherwise, return to **Substep** $2.\,b\,(iii)$.

Substep $2.\,b\,(v)$: Modify the internal offer $\beta_i^{\dagger}$ to create $\tilde{\beta}_i^{\dagger} = \{p_n^{\dagger\to\ddagger}, \mu_i^{\dagger} + \mu_n^{\dagger\to\ddagger}\}$ and $\tilde{\beta}_{I+i}^{\dagger} = \{p_i^{\dagger} - p_n^{\dagger\to\ddagger}, \mu_i^{\dagger}\}$. Set $i = i+1$ and $n = n+1$. **Substep** $2.\,b\,(vi)$: If $i = I$, terminate; otherwise, return to **Substep** $2.\,b\,(v)$.

Substep $2.\,b\,(vii)$: Arrange the internal offers at † after modification $\{\tilde{\beta}^{\dagger}\}$ in the order of decreasing offer prices to obtain $\tilde{\mu}_1^{\dagger} \geq \tilde{\mu}_2^{\dagger} \geq ... \geq \tilde{\mu}_{\tilde{I}}^{\dagger}$.

If $I \geq N$, then N of the offers are modified to create $2N$ offers, and $I - N$ offers remain unchanged; therefore, $\tilde{I} = I + N$. If $I < N$, then the I offers are modified to create $2I$ modified offers, and each initial offer is modified; therefore, $\tilde{I} = 2I$.

The matching of the internal offers at † with the interface offers is a preparatory step prior to the modification of the internal offers at † by the interface offers. The rationale of the rearrangement of the order of the internal offers at † from the highest offer price to the lowest offer price, and that of the interface offers from the lowest price difference to the highest price difference, is to ensure that the offer price of the expensive offers are reduced as small as possible and the expensive offers do not get cleared after the modification. Had the internal offer with the highest offer price been matched by the interface offer with the highest price difference, then the offer price of the modified internal offer could have decreased significantly, and such an expensive internal offer could have cleared due to its significantly reduced price after its modification.

The incorporation of the interface offers in CTS renders CTS a procedure that harnesses market forces to determine the interchange evaluation. The rationale behind the particular way of modification of the internal offers at † is provided for two distinct cases of interface exchanges.

For the case that the interface exchange is from ‡ to †, only the interface offers $\{\beta^{\ddagger\rightarrow\dagger}\}$, which are submitted to transact power from ‡ to †, are considered. The match of the interface offer $\beta_c^{\ddagger\rightarrow\dagger} = \{p_c^{\ddagger\rightarrow\dagger}, \mu_c^{\ddagger\rightarrow\dagger}\}$ with the internal offer at † $\beta_i^{\dagger} = \{p_i^{\dagger}, \mu_i^{\dagger}\}$ translates as $p_c^{\ddagger\rightarrow\dagger}$ MW of withdrawal from ‡ at the price $(\mu_i^{\dagger} - \mu_c^{\ddagger\rightarrow\dagger})$ $\$/MWh$ and $p_c^{\ddagger\rightarrow\dagger}$ MW of injection to † at the price $\mu_i^{\dagger}$ $\$/MWh$, in line with the condition of the interface offer of at least $\mu_c^{\ddagger\rightarrow\dagger}$ price difference between the two buses. The modification of the internal offer $\beta_i^{\dagger} = \{p_i^{\dagger}, \mu_i^{\dagger}\}$ by the interface offer $\beta_c^{\ddagger\rightarrow\dagger} = \{p_c^{\ddagger\rightarrow\dagger}, \mu_c^{\ddagger\rightarrow\dagger}\}$ creates the modified internal offer $\tilde{\beta}_i^{\dagger} = \{p_c^{\ddagger\rightarrow\dagger}, \mu_i^{\dagger} - \mu_c^{\ddagger\rightarrow\dagger}\}$

at †. The price and power values of the created fictitious internal offer at † are identical to the price and power values by which the interface offerer wishes to withdraw at ‡. Therefore, the fictitious creation of the identical offer at † obviates the need to transact that power from ‡, and acts as a barrier to the interface offerer that wishes to take advantage of the price difference across the interface.

For the case that the interface exchange is from † to ‡, only the interface offers submitted to transact power from † to ‡ are considered. The match of an interface offer $\beta_n^{\dagger\rightarrow\ddagger} = \{p_n^{\dagger\rightarrow\ddagger}, \mu_n^{\dagger\rightarrow\ddagger}\}$ with the internal offer at † $\beta_i^{\dagger} = \{p_i^{\dagger}, \mu_i^{\dagger}\}$ translates as $p_n^{\dagger\rightarrow\ddagger}$ MW of withdrawal from † at the price $(\mu_i^{\dagger})$ $\$/MWh$ and $p_n^{\dagger\rightarrow\ddagger}$ MW of injection to ‡ at the price $\mu_i^{\dagger} + \mu_n^{\dagger\rightarrow\ddagger}$ $\$/MWh$, in line with the condition of the interface offer of at least $\phi_n^{\dagger\rightarrow\ddagger}$ price difference between the two buses. The modification of the internal offer $\beta_i^{\dagger} = \{p_i^{\dagger}, \mu_i^{\dagger}\}$ by the interface offer $\beta_n^{\dagger\rightarrow\ddagger} = \{p_n^{\dagger\rightarrow\ddagger}, \mu_n^{\dagger\rightarrow\ddagger}\}$ creates the modified internal offer $\tilde{\beta}_i^{\dagger} = \{p_n^{\dagger\rightarrow\ddagger}, \mu_i^{\dagger} + \mu_n^{\dagger\rightarrow\ddagger}\}$ at †. The price and power values of the created fictitious internal offer at † are identical to the price and power values by which the interface offerer wishes to inject at ‡. Therefore, the fictitious creation of the identical offer at † obviates the need to transact that power to ‡, and acts as a barrier to the interface offerer that wishes to take advantage of the price difference across the interface.

For both cases, only the internal offers at † are modified, and the internal offers at ‡ remain unchanged. This is to ensure that the modification can act as a barrier to the interface offerer that wishes to take advantage of the price difference across the interface. If the internal offers at both † and ‡ were modified, then the price difference across the interface would remain the same. Since the interface offerer is only interested in the price difference across the interface, the modifications of the internal offers at both † and ‡ have no impact on the interface offerer.

3.4 Step 3: Determination of the Interface Exchange Amount

The CTS model that is accommodated in the economic layer of the CTS framework determines the interface exchange amount as a by-product of the solution of yet another ED problem. This ED problem assumes the hypothetical aggregation of the two interconnected IGOs to form a single system, and is solved based on the modified internal offers at † $\{\tilde{\beta}^{\dagger}\}$, the internal offers ‡ $\{\beta^{\ddagger}\}$, and the combined forecasted intra-hour demands at † and ‡, $d^{\dagger} + d^{\ddagger}$. In order to formulate the ED problem, we again use the variable $a_{j}^{\ddagger}$ $(\tilde{a}_{\tilde{i}}{}^{\dagger})$, which represents the cleared real power in MW from the j^{th} ($\tilde{i}^{th}$) internal offer. Hence, $a^{\ddagger}_{j}$, $j = 1, 2, ...J$ and $\tilde{a}_{\tilde{i}}{}^{\dagger}$, $\tilde{i} = 1, 2, ...\tilde{I}$ serve as the *optimization variables* in the *ED* problem formulation.

The *ED* problem for the hypothetically aggregated system can be formulated as follows:

$$
\begin{aligned}
\min_{\tilde{a}^{\dagger}_{\tilde{i}},\, a^{\ddagger}_{j}} \quad & \sum_{\tilde{i}=1}^{\tilde{I}} (\tilde{a}^{\dagger}_{\tilde{i}})\,(\tilde{\mu}^{\dagger}_{\tilde{i}}) \; + \; \sum_{j=1}^{J} (a^{\ddagger}_{j})\,(\mu^{\ddagger}_{j}) \\
s.t. \quad & \sum_{\tilde{i}=1}^{\tilde{I}} \tilde{a}_{\tilde{i}} + \sum_{j=1}^{J} a_{j} \; - \; (d^{\dagger} + d^{\ddagger}) = 0 \quad \longleftrightarrow \; \tilde{\chi} \\
& \tilde{a}_{\tilde{i}} - \tilde{p}_{\tilde{i}} \leq 0 \quad \tilde{i} = 1, 2, ...\tilde{I} \\
& a_{j} - p_{j} \leq 0 \quad j = 1, 2, ...J
\end{aligned}
\tag{3.2}
$$

The optimal decision variables $\tilde{a}^{\dagger\,*}_{\tilde{i}}$ $\tilde{i} = 1, 2, ...\tilde{I}$ and $a^{\ddagger\,*}_{j}$ $j = 1, 2, ...J$ indicate the *MW* of power cleared from the internal offers $\tilde{i} = 1, 2, ...\tilde{I}$ and $j = 1, 2, ...J$ by solving (3.2). Further, we denote the dual variable associated with the equality constraint at the optimum as $\tilde{\chi}^{*}$. $\tilde{\chi}^{*}$ is the marginal price to serve the last *MW* of the aggregated demand $d^{\dagger} + d^{\ddagger}$.

The total power sold by the internal offerers at $\dagger$ can be represented as $\sum_{\tilde{i}=1}^{\tilde{I}} \tilde{a}^{\dagger\,*}_{\tilde{i}}$. Similarly, the total power sold by the internal offerers of $\ddagger$ can be represented as

$\sum_{j=1}^{J} a_j^{\ddagger *}$. Referring to our convention that if the interface exchange is from ‡ to †, then $f_\ell > 0$, and if the interface exchange is from † to ‡, then $f_\ell < 0$, we can write the following set of equations for the interface exchange

$$\begin{aligned} \sum_{j=1}^{J} a_j^{\ddagger *} &= d^{\ddagger} + f_\ell \\ \sum_{\tilde{i}=1}^{\tilde{I}} \tilde{a}_{\tilde{i}}^{\dagger *} + f_\ell &= d^{\dagger} \end{aligned} \tag{3.3}$$

Hence, after the completion of this step, the interface exchange amount f_ℓ is determined. The determined interface exchange amount is passed from the CTS model accommodated in the economic layer to the interface model accommodated in the physical layer.

As explained in Appendix B, tie-line optimization (TLO) is a *conceptual construct* to determine the interface exchange amount between † and ‡. TLO assumes that the two interconnected systems are hypothetically aggregated to form a single *super-system*. The *super-system* is operated by a *super-IGO* that overtakes the functions of both IGOs and so has the knowledge of the offers and the intra-hour demands of the two IGOs. In TLO, the *super-IGO* solves an ED problem for the *super-system* based on the internal offers and intra-hour demands at † and ‡. This step can be considered as a TLO-like procedure, as similar to the TLO, an ED problem is solved for the hypothetically aggregated system. However, instead of the initial internal offers at †, the modified internal offers are used, which distinguishes this step from the TLO.

In the case that the interface exchange is from ‡ to †, the problem formulation (3.2) implies that the total cleared power from the internal offers at † increases with the modification because the modified offers are at prices equal to or below the initial internal offers at †. Therefore, the CTS procedure clears more internal offers at † than under the condition that the initial internal offers of † were used instead of the modified internal offers at †. Since TLO differs from CTS only by the use of the original internal offers at † instead of the modified internal offers at †, CTS procedure clears more internal offers at † than TLO does when the interface exchange is from ‡ to †. Such an increase of the cleared internal offers at † is counter-intuitive because for the case that the interface exchange is from ‡ to †, it is desired to meet the forecasted intra-hour demand with the cheaper internal offers at ‡ instead of the more expensive internal offers at †.

In the case that the interface exchange is from † to ‡, the problem formulation

(3.2) implies that the total cleared power from the internal offers at † decreases with the modification because the modified offers are at prices greater than or equal to the initial internal offers at †. Therefore, the CTS procedure clears more internal offers at ‡ than under the condition that the initial internal offers of † were used instead of the modified internal offers at †. Since TLO differs from CTS only by the use of the original internal offers at † instead of the modified internal offers at †, the CTS procedure clears more internal offers at ‡ than TLO does when the interface exchange is from † to ‡. Such an increase of the cleared internal offers at ‡ is again counter-intuitive because, for the case that the interface exchange is from † to ‡, it is desired to meet the forecasted intra-hour demand with the cheaper internal offers at † instead of the more expensive internal offers at ‡.

The counter-intuitive increase in the cleared internal offers at † when the interface exchange is from ‡ to †, and the counter-intuitive increase in the cleared internal offers at ‡ when the interface exchange is from † to ‡, will help us understand why the total payments obtained via CTS is higher than or equal to those obtained with TLO in the next section.

3.5 Step 4: Clearance of the Internal Offers and the Interface Offers

In the previous step, the CTS model has determined the total power that must be cleared from the internal offers at †, $\sum_{\tilde{i}=1}^{\tilde{I}} \tilde{a}_i^{\dagger\,*}$, and the internal offers at ‡, $\sum_{j=1}^{J} a_j^{\ddagger*}$. In this step, the original internal offers submitted to the bus of each system are cleared such that the total power that must be cleared in that bus is met in the most economical way. Namely, the original internal offers at † $\{\beta^{\dagger}\}$ are cleared in the most economical way such that the total cleared power at † is $\sum_{\tilde{i}=1}^{\tilde{I}} \tilde{a}_i^{\dagger\,*}$. Similarly, the original internal offers at ‡ $\{\beta^{\ddagger}\}$ are cleared in the most economical way such that the total cleared power at † is $\sum_{j=1}^{J} a_j^{\ddagger*}$.

Once the cleared internal offers at † (‡) are determined, we evaluate the marginal price of electricity to serve an additional MW of demand at †(‡), which we denote by $\chi_{\dagger}$ ($\chi_{\ddagger}$). The CTS model determines which interface offers are cleared based on $\chi_{\dagger}$ and $\chi_{\ddagger}$. If the interface exchange is from ‡ to †, then the interface offers with price differences such that $\mu_c^{\ddagger\to\dagger} \leq \mu_{\dagger} - \mu_{\ddagger}$ are cleared, and those with $\mu_c^{\ddagger\to\dagger} > \mu_{\dagger} - \mu_{\ddagger}$ are not cleared. If the interface exchange is from † to ‡, then the interface offers with price differences such that $\mu_n^{\dagger\to\ddagger} \leq \mu_{\ddagger} - \mu_{\dagger}$ are cleared, and those with $\mu_n^{\dagger\to\ddagger} > \mu_{\ddagger}$

- $\mu_\dagger$ are not cleared.

The intra-hour demands of the two interconnected IGOs are met with the original internal offers because the modified internal offers are basically constructs of CTS and not actual submitted offers. When the interface exchange is from ‡ to †, we refer to our observation in Step 3 that CTS clears more internal offers at † than does TLO. The higher amount of power cleared at † will be cleared from the initial and more expensive internal offers at †, which causes the total payments incurred in CTS to be greater than or equal to those obtained with TLO.

When the interface exchange is from † to ‡, we refer to our observation in Step 3 that CTS clears more internal offers at ‡ than does TLO. The higher amount of power cleared at ‡ will be cleared from the more expensive initial internal offers at ‡, which causes the total payments incurred in CTS to be greater than or equal to those obtained with TLO. When the interface exchange is from † to ‡, the modified internal offers at † are more expensive than the initial internal offers at †; however, the fictitious creation of the expensive internal offers at † precludes the utilization of these cheap initial internal offers at † and makes the CTS model resort to the internal offers at ‡, which requires greater total payments. Therefore, the total payments obtained via CTS are greater than or equal to the total payments obtained via TLO when the interface exchange is from † to ‡ as well.

When the interface exchange is from ‡ to †, only the interface offers with price differences that are less than or equal to $\mu_{\dagger} - \mu_{\ddagger}$ are cleared. Similarly, when the interface exchange is from † to ‡, only the interface offers with price differences less than or equal to $\mu_{\ddagger} - \mu_{\dagger}$ are cleared. Therefore, an interface offerer who wishes to clear the market must necessarily submit the interface offer with a smaller price difference to be successful.

3.6 Concluding Remarks and Summary

In this chapter, we have discussed CTS. We have tailored the framework constructed in Chapter 2 for CTS, and constructed the power system and internal market models of the IGOs, as well as the interface model. We have described the data/information that must be collected by the IGOs through the respective models. Since each IGO wants to maintain the confidentiality of its data/information, we have decided on the share of data/information for the execution of CTS so as to allow the CTS to run with minimum data/information exchange between the models of the two interconnected IGOs. We have provided an analytical underpinning of CTS, and explained the procedures of CTS in detail. In the first step of CTS, the interface exchange direction is determined so as to ensure that it is from the bus with lower marginal price to the bus with higher marginal price. Next, we have shown the substeps for the modification of the internal offers at † by the interface offers for the two cases of interface exchange direction. In the third step of CTS, we have described the determination of the interface exchange amount. Finally, we have shown the clearance of the internal offers at † and ‡, as well as the interface offers. We have explained why the total payments obtained via CTS are greater than or equal to those obtained via TLO. In Chapter 4, we will provide illustrative examples of the execution of CTS. We will also identify the factors that influence the duration of the execution of the steps of CTS in Chapter 5, and will work toward shortening the coordination period.

CHAPTER 4
ILLUSTRATIVE EXAMPLES OF THE EXECUTION OF CTS

The discussion of CTS in Chapter 3 provided an analytical underpinning and a detailed description of the procedural steps of CTS. In this chapter, we illustrate the execution of these procedural steps by the utilization of the CTS framework described in Section 3.1. Throughout the chapter, we adopt the time convention introduced in Section 2.1, and illustrate the execution of CTS for the coordination period $[k]\big|_h$. We introduce two sample data sets and input the data to the appropriate models accommodated in the CTS framework. We illustrate the execution of the steps described in Section 3.2–3.5 all the while referring to the appropriate models of the CTS framework, as well as the required interactions between the models. We obtain the interface exchange, cleared internal offers, cleared interface offers, and the total payments for each data set. We conduct sensitivity analyses and study the dependence of the interface exchange and the total payments on the submitted internal and interface offers.

In Section 4.1, we introduce Data Set 1 and Data Set 2. In Section 4.2, we illustrate the execution of the procedural steps of CTS for Data Set 1. We devote Section 4.3 to the introduction of Data Set 2, and the illustration of the execution of the procedural steps of CTS for Data Set 2. We conduct various sensitivity analyses in Section 4.4. Finally, we present our concluding remarks and summarize the chapter in Section 4.5.

4.1 Presentation of the Data Sets

In this section, we present two sample data sets, Data Set 1 and Data Set 2, which serve as inputs to the respective models that get accommodated in the CTS framework. The two data sets are used to illustrate both the execution of CTS in Sections 4.2 and 4.3, and the dependence of the interface exchange and the total payments on a change in the internal offers or interface offers. Each data set contains the internal offers at † and ‡, the intra-hour demands at † and ‡, and the interface offers. Referring to the CTS framework developed in Section 3.1, the internal offers and the intra-hour demand at † (‡) serve as inputs to the IGO^M (IGO^F) internal market model that get accommodated in the economic layer of the framework. The interface offers serve as inputs to the CTS model that get accommodated the economic layer of the framework.

Data Set 1

Data Set 1 includes the internal offers at † and ‡, the intra-hour demands at † and ‡, and the interface offers. Table 4.1 lists the internal offers that are submitted to sell power at the bus † of IGO^M. We consider the intra-hour demand at the bus † of IGO^M as $d^\dagger = 230\, MW$. Table 4.2 lists the internal offers that are submitted to sell power at the bus ‡ of IGO^F. We consider the intra-hour demand at the bus ‡ of IGO^F as $d^\ddagger = 220\, MW$. In Table 4.3, we provide the interface offers that are submitted to transact power from the bus ‡ of IGO^F to the bus † of IGO^M. Finally, Table 4.4 contains the interface offers that are submitted to transact power from the bus † of IGO^M to the bus ‡ of IGO^F.

Table 4.1: Data Set 1 internal offers at †

i	$\beta_i^\dagger = \{p_i^\dagger\,,\,\mu_i^\dagger\}$
1	$\{170\,,\,12.00\}$
2	$\{150\,,\,11.00\}$
3	$\{120\,,\,10.00\}$
4	$\{120\,,\,9.00\}$
5	$\{70\,,\,8.00\}$

Table 4.2: Data Set 1 internal offers at ‡

j	$\beta_j^\ddagger = \{p_j^\ddagger\,,\,\mu_j^\ddagger\}$
1	$\{150\,,\,8.00\}$
2	$\{130\,,\,9.00\}$
3	$\{100\,,\,10.00\}$
4	$\{130\,,\,10.50\}$
5	$\{120\,,\,12.00\}$
6	$\{100\,,\,13.00\}$

Table 4.3: Data Set 1 ‡ to † interface offers

c	$\beta_c^{\ddagger\to\dagger} = \{p_c^{\ddagger\to\dagger}\,,\, \mu_c^{\ddagger\to\dagger}\}$
1	$\{20\,,\, 0.75\}$
2	$\{30\,,\, 1.00\}$
3	$\{40\,,\, 1.25\}$
4	$\{30\,,\, 1.50\}$

Table 4.4: Data Set 1 † to ‡ interface offers

n	$\beta_n^{\dagger\to\ddagger} = \{p_n^{\dagger\to\ddagger}\,,\, \mu_n^{\dagger\to\ddagger}\}$
1	$\{30\,,\, 0.50\}$
2	$\{20\,,\, 0.75\}$
3	$\{40\,,\, 1.50\}$

Data Set 2

In addition to the Data Set 1, we provide another data set, Data Set 2, which will allow us to illustrate both the execution of CTS, and the dependence of the interface exchange and the total payments on a change in the internal or interface offers. In Table 4.5, we list the internal offers that are submitted to sell power at the bus † of IGO^M. We consider the intra-hour demand at the bus † of IGO^M as $d^{\dagger} = 150\, MW$. Table 4.6 contains the internal offers that are submitted to sell power at the bus ‡ of IGO^F. We consider the intra-hour demand at the bus ‡ of IGO^F as $d^{\ddagger} = 270\, MW$. Table 4.7 lists the interface offers that are submitted to transact power from the bus ‡ of IGO^F to the bus † of IGO^M. Finally, Table 4.8 lists the interface offers that are submitted to transact power from the bus † of IGO^M to the bus ‡ of IGO^F.

Table 4.5: Data Set 2 internal offers at †

i	$\beta_i^{\dagger} = \{p_i^{\dagger}\,,\, \mu_i^{\dagger}\}$
1	$\{130\,,\, 11.00\}$
2	$\{140\,,\, 10.00\}$
3	$\{200\,,\, 9.00\}$
4	$\{60\,,\, 8.00\}$

Table 4.6: Data Set 2 internal offers at ‡

j	$\beta_j^{\ddagger} = \{p_j^{\ddagger}, \mu_j^{\ddagger}\}$
1	$\{80, 8.00\}$
2	$\{90, 9.00\}$
3	$\{110, 10.00\}$
4	$\{140, 11.00\}$
5	$\{100, 12.00\}$

Table 4.7: Data Set 2 ‡ to † interface offers

c	$\beta_c^{\ddagger\to\dagger} = \{p_c^{\ddagger\to\dagger}, \mu_c^{\ddagger\to\dagger}\}$
1	$\{30, 0.75\}$
2	$\{20, 1.00\}$
3	$\{30, 1.50\}$

Table 4.8: Data Set 2 † to ‡ interface offers

n	$\beta_n^{\dagger\to\ddagger} = \{p_n^{\dagger\to\ddagger}\,,\,\mu_n^{\dagger\to\ddagger}\}$
1	$\{30\,,\,0.50\}$
2	$\{50\,,\,0.75\}$
3	$\{40\,,\,1.25\}$
4	$\{30\,,\,1.50\}$
5	$\{40\,,\,1.75\}$

4.2 Illustrative Example 1

In this section, we use the Data Set 1 presented in Section 4.1 to demonstrate the steps of the CTS as explained in Chapter 3. The description of CTS in Chapter 3.1 shows us that before the two interconnected IGOs can begin the execution of CTS they need to collect the required data/information, which serve as inputs to the models that get accommodated in the CTS framework. The internal offers at † listed in Table 4.1 and the intra-hour demand at † serve as inputs to the IGO^M internal market model in the economic layer, and the internal offers at ‡ listed in Table 4.2 and the intra-hour demand at ‡ serve as inputs to the IGO^F internal market model in the economic layer. The † to ‡ interface offers in Table 4.3 and the ‡ to † interface offers in Table 4.4 are provided as inputs to the CTS model in the economic layer.

The data/information need to be passed from the respective models to the CTS model for the execution of CTS. The IGO^M (IGO^F) internal market model passes the internal offers at † (‡) and the intra-hour demand at † (‡) to the CTS model. The required data/information are collected in the CTS model.

<u>CTS Step 1: Determination of the interface exchange direction</u>

The first step of CTS is the determination of the interface exchange direction. Following the analysis described in Section 3.2, the CTS model solves an ED problem for each IGO that does not take into account the interface exchange. The optimization variable of the ED problem for IGO^M is $\acute{a}_i^{\dagger}$, which represents the cleared real power in MW from the i^{th} submitted internal offer at † for the ED problem described in this step. The ED problem solved for IGO^M is formulated in (4.1).

$$\begin{aligned} &\min_{\acute{a}_i^{\dagger}} \quad \sum_{i=1}^{I} (\acute{a}_i^{\dagger})\,(\mu_i^{\dagger}) \\ s.t.\ &\sum_{i=1}^{I} \acute{a}_i^{\dagger} - d^{\dagger} = 0 \quad \longleftrightarrow \quad \acute{\chi}^{\dagger} \\ &\acute{a}_i^{\dagger} - p_i^{\dagger} \leq 0 \quad i = 1, 2, ...I \end{aligned} \tag{4.1}$$

where $d^{\dagger} = 230\ MW$, and $p_i^{\dagger}$ and $\mu_i^{\dagger}$ of the respective internal offer i at † are listed in Table 4.1, for $i = 1, .., I$ and $I = 5$.

The objective of the optimization problem is to meet the demand $d^{\dagger} = 230\ MW$ in the cheapest way from the internal offers at † that are listed in Table 1. By observation, we can solve (4.1) by clearing the internal offers at Table 4.1 in the order of lowest to highest offer prices. Table 4.9 indicates the cleared internal offer amounts to meet the demand $d^{\dagger} = 230\ MW$.

Table 4.9: Clearance of the internal offers at † for CTS Step 1

i	$\beta_i^{\dagger} = \{p_i^{\dagger}, \mu_i^{\dagger}\}$	$\acute{a}_i^{\dagger *}$
1	$\{170, 12.00\}$	0
2	$\{150, 11.00\}$	0
3	$\{120, 10.00\}$	40
4	$\{120, 9.00\}$	120
5	$\{70, 8.00\}$	70

The last internal offer at † that was cleared was $\beta_3^{\dagger}$. A 1 MW increase in demand will be met by clearing an additional 1 MW from the offer $\beta_3^{\dagger} = \{120, 10\}$, hence the marginal price of electricity at † for the ED problem defined in (4.1) is $\acute{\chi}^{\dagger} = 10$

$\$/MWh$.

While the CTS model solves (4.1) for IGO^M, it simultaneously solves the exact same ED problem for IGO^F. The optimization variable of the ED problem for IGO^F is $\acute{a}_j^{\ddagger}$, which represents the cleared real power in MW from the j^{th} submitted internal offer for the ED problem described in this step. The ED problem for IGO^F is formulated in (4.2).

$$\begin{aligned} & \min_{\acute{a}_j^{\ddagger}} \quad \sum_{j=1}^{J} (\acute{a}_j^{\ddagger})\,(\mu_j^{\ddagger}) \\ s.t. \; & \sum_{j=1}^{J} \acute{a}_j^{\ddagger} - d^{\ddagger} = 0 \quad \longleftrightarrow \; \acute{\chi}^{\ddagger} \\ & \acute{a}_j^{\ddagger} - p_j^{\ddagger} \leq 0 \quad j = 1, 2, ...J \end{aligned} \tag{4.2}$$

where $d^{\ddagger} = 220\ MW$, and $p_j^{\ddagger}$ and $\mu_j^{\ddagger}$ of the respective internal offer j at $\ddagger$ are as listed in Table 4.2, for $j = 1, .., J$ and $J = 6$. The objective of the optimization problem is to meet the demand $d^{\ddagger} = 220\ MW$ in the cheapest way from the internal offers at $\ddagger$ that are listed in Table 4.2. By observation, we can solve (4.2) by clearing the internal offers at Table 4.2 in the order of lowest to highest offer prices. Table 4.10 indicates the cleared internal offer amounts to meet the demand $d^{\ddagger} = 220\ MW$.

Table 4.10: Clearance of the internal offers at ‡ for CTS Step 1

j	$\beta_j^{\ddagger} = \{p_j^{\ddagger}\,,\,\mu_j^{\ddagger}\}$	$\acute{a}_j^{\ddagger *}$
1	$\{150\,,\,8.00\}$	150
2	$\{130\,,\,9.00\}$	70
3	$\{100\,,\,10.00\}$	0
4	$\{130\,,\,10.50\}$	0
5	$\{120\,,\,12.00\}$	0
6	$\{100\,,\,13.00\}$	0

The last internal offer at ‡ that was cleared was $\beta_2^{\ddagger}$. A 1 MW increase in demand will be met by clearing an additional 1 MW from the offer $\beta_2^{\ddagger} = \{130\,,\,9.00\}$, hence the marginal price of electricity at ‡ for the ED problem defined in (4.2) is $\acute{\chi}^{\ddagger} = 9.00$ $\$/MWh$. The CTS procedure allows the interface exchange to be only from the bus with the lower marginal price to the bus with the higher marginal price. Since $\acute{\chi}^{\ddagger} = 9.00$ $\$/MWh$ and $\acute{\chi}^{\dagger} = 10.00$ $\$/MWh$, the interface exchange direction is from the bus ‡ of IGO^F to the bus † of IGO^M.

CTS Step 2: Modification of the internal offers at †

The interface exchange direction that was determined in the previous step of CTS designates how the internal offers at the bus † are modified by the interface offers.

Since the interface exchange is from the bus ‡ of IGO^F to the bus † of IGO^M, the ‡ to † interface offers are used to modify the internal offers at †, and the † to ‡ interface offers are not considered. Following our discussion in Section 3.3, we arrange the internal offers at † in descending order of their offer prices in Table 4.11. Further, we arrange the ‡ to † interface offers in the ascending order of their price differences in Table 4.12. We follow the substeps indicated in Section 3.3 and modify the internal offers as follows:

$\beta_1^\dagger = \{170\,,\ 12.00\}$ is modified by $\beta_1^{\ddagger\to\dagger} = \{20\,,\ 0.75\}$ to create $\tilde{\beta}_1^\dagger = \{20, 11.25\}$ and $\tilde{\beta}_6^\dagger = \{150, 12.00\}$.

$\beta_2^\dagger = \{150\,,\ 11.00\}$ is modified by $\beta_2^{\ddagger\to\dagger} = \{30\,,\ 1.00\}$ to create $\tilde{\beta}_2^\dagger = \{30, 10.00\}$ and $\tilde{\beta}_7^\dagger = \{120, 11.00\}$.

$\beta_3^\dagger = \{120\,,\ 10.00\}$ is modified by $\beta_3^{\ddagger\to\dagger} = \{40\,,\ 1.25\}$ to create $\tilde{\beta}_3^\dagger = \{40, 8.75\}$ and $\tilde{\beta}_8^\dagger = \{80, 10.00\}$.

$\beta_4^\dagger = \{120\,,\ 9.00\}$ is modified by $\beta_4^{\ddagger\to\dagger} = \{30\,,\ 1.50\}$ to create $\tilde{\beta}_4^\dagger = \{30, 7.50\}$ and $\tilde{\beta}_9^\dagger = \{90, 9.00\}$.

$\beta_5^\dagger = \{70\,,\ 8.00\}$ is not modified because all four interface offers have been used to modify the previous four internal offers at †. Hence, $\tilde{\beta}_5^\dagger = \{70, 8.00\}$.

Table 4.11: Internal offers at † arranged in descending order of offer prices

i	$\beta_i^{\dagger} = \{p_i^{\dagger}\,,\, \mu_i^{\dagger}\}$
1	$\{170\,,\, 12.00\}$
2	$\{150\,,\, 11.00\}$
3	$\{120\,,\, 10.00\}$
4	$\{120\,,\, 9.00\}$
5	$\{70\,,\, 8.00\}$

Table 4.12: ‡ to † interface offers arranged in ascending order of offered price differences

c	$\beta_c^{\ddagger\to\dagger} = \{p_c^{\ddagger\to\dagger}\,,\, \mu_c^{\ddagger\to\dagger}\}$
1	$\{20\,,\, 0.75\}$
2	$\{30\,,\, 1.00\}$
3	$\{40\,,\, 1.25\}$
4	$\{30\,,\, 1.50\}$

<u>CTS Step 3: Determination of the interface exchange amount</u>

After the modified internal offers at †, $\{\tilde{\beta}^{\dagger}\}$, are obtained in the previous step, we follow the discussion in Section 3.4 and determine the interface exchange amount as a by-product of the solution of yet another ED problem for the hypothetical aggregation of the two interconnected IGOs as a single system based on the modified internal offers at † $\{\tilde{\beta}^{\dagger}\}$, the internal offers ‡ $\{\beta^{\ddagger}\}$, and the combined forecasted intra-hour demands at † and ‡, $d^{\dagger} + d^{\ddagger}$.

The ED problem for the hypothetically aggregated system can be formulated as follows:

$$
\begin{aligned}
&\min_{\tilde{a}_{\tilde{i}}^{\dagger},\, a_j^{\ddagger}} \quad \sum_{\tilde{i}=1}^{\tilde{I}} (\tilde{a}_{\tilde{i}}^{\dagger})\,(\tilde{\mu}_{\tilde{i}}^{\dagger}) \;+\; \sum_{j=1}^{J} (a_j^{\ddagger})\,(\mu_j^{\ddagger}) \\
&s.t. \quad \sum_{\tilde{i}=1}^{\tilde{I}} \tilde{a}_{\tilde{i}} + \sum_{j=1}^{J} a_j \;-\; (\bar{d}^{\dagger} + \bar{d}^{\ddagger}) \;=\; 0 \quad \longleftrightarrow \quad \tilde{\chi} \\
&\qquad \tilde{a}_{\tilde{i}} - \tilde{p}_{\tilde{i}} \leq 0 \quad \tilde{i} = 1, 2, ...\tilde{I} \\
&\qquad a_j - p_j \leq 0 \quad j = 1, 2, ...J
\end{aligned}
\tag{4.3}
$$

The optimal decision variables $\tilde{a}_{\tilde{i}}^{\dagger\,*}$ $\tilde{i} = 1, 2, ...\tilde{I}$ and $a_j^{\ddagger\,*}$ $j = 1, 2, ...J$ indicate the MW of power cleared from the internal offers $\tilde{i} = 1, 2, ...\tilde{I}$ and $j = 1, 2, ...J$. In order to illustrate the solution of the optimization problem (4.3), we form a table that contains the modified internal offers at † $\{\tilde{\beta}^{\dagger}\}$ and the internal offers at ‡ $\{\beta^{\ddagger}\}$ arranged in ascending order of their offer prices. Given that $d^{\dagger} = 230\,MW$ and $d^{\ddagger} = 220\,MW$, we select the offers such that $d^{\dagger} + d^{\ddagger} = 450\,MW$ is met from the cheapest $450\,MW$ offers. Table 4.13 lists the cleared modified internal offer amounts at † and the cleared internal offer amounts at ‡ to meet the demand $d^{\dagger} + d^{\ddagger} = 450\,MW$.

The total power sold by the offerers at † can be represented by $\sum_{\tilde{i}=1}^{\tilde{I}} \tilde{a}_{\tilde{i}}^{\dagger *}$. From Table 4.13, the total power sold by the internal offerers at † is $220\,MW$. Similarly, the total power sold by the internal offerers at ‡ can be represented as $\sum_{j=1}^{J} a_{j}^{\ddagger *}$. Also from Table 4.13, the total power sold by the internal offerers at ‡ is $230\,MW$. Plugging the values $d^{\dagger} = 230\,MW$, $d^{\ddagger} = 220\,MW$, $\sum_{\tilde{i}=1}^{\tilde{I}} \tilde{a}_{\tilde{i}}^{\dagger *} = 220\,MW$, and $\sum_{j=1}^{J} a_{j}^{\ddagger *} = 230\,MW$ in (4.4) as explained in Section 3.3

$$\begin{aligned} \sum_{j=1}^{J} a_{j}^{\ddagger *} &= d^{\ddagger} + f_{\ell} \\ \sum_{\tilde{i}=1}^{\tilde{I}} \tilde{a}_{\tilde{i}}^{\dagger *} + f_{\ell} &= d^{\dagger} \end{aligned} \tag{4.4}$$

we obtain $f_{\ell} = 10\,MW$.

Table 4.13: Clearance of the modified internal offers at † and the internal offers at ‡ for the solution of (4.3)

$\beta = \{p\,,\,\mu\}$	a^*
$\tilde{\beta}_4^\dagger = \{30\,,\,7.5\}$	$\tilde{a}_4^{\dagger^*} = 30$
$\tilde{\beta}_5^\dagger = \{70\,,\,8\}$	$\tilde{a}_5^{\dagger^*} = 70$
$\beta_1^\ddagger = \{150\,,\,8\}$	$a_1^{\ddagger^*} = 150$
$\tilde{\beta}_3^\dagger = \{40\,,\,8.75\}$	$\tilde{a}_3^{\dagger^*} = 40$
$\tilde{\beta}_9^\dagger = \{90\,,\,9\}$	$\tilde{a}_9^{\dagger^*} = 80$
$\beta_2^\ddagger = \{130\,,\,9\}$	$a_2^{\ddagger^*} = 80$
$\tilde{\beta}_2^\dagger = \{30\,,\,10\}$	$\tilde{a}_2^{\dagger^*} = 0$
$\tilde{\beta}_8^\dagger = \{80\,,\,10\}$	$\tilde{a}_8^{\dagger^*} = 0$
$\beta_3^\ddagger = \{100\,,\,10\}$	$a_3^{\ddagger^*} = 0$
$\beta_4^\ddagger = \{130\,,\,10.5\}$	$a_4^{\ddagger^*} = 0$
$\tilde{\beta}_7^\dagger = \{120\,,\,11\}$	$\tilde{a}_7^{\dagger^*} = 0$
$\tilde{\beta}_1^\dagger = \{20\,,\,11.25\}$	$\tilde{a}_1^{\dagger^*} = 0$
$\tilde{\beta}_6^\dagger = \{150\,,\,12\}$	$\tilde{a}_6^{\dagger^*} = 0$
$\beta_5^\ddagger = \{120\,,\,12\}$	$a_5^{\ddagger^*} = 0$
$\beta_6^\ddagger = \{100\,,\,13\}$	$a_6^{\ddagger^*} = 0$

CTS Step 4: Clearance of the internal offers and the interface offers

In this step, the CTS model clears the original internal offers at † so that $\sum_{\tilde{i}=1}^{\tilde{I}} \tilde{a}_{\tilde{i}}^{\dagger *} = 220\,MW$ is met in the most economical way. Table 4.14 lists the internal offers at † and the cleared power amount from each offer in order to satisfy $220\,MW$.

Table 4.14: Clearance of the original internal offers at † and ‡ by CTS

i	$\beta_i^{\dagger} = \{p_i^{\dagger}\,,\, \mu_i^{\dagger}\}$	$a_i^{\dagger *}$
1	$\{170\,,\, 12.00\}$	0
2	$\{150\,,\, 11.00\}$	0
3	$\{120\,,\, 10.00\}$	30
4	$\{120\,,\, 9.00\}$	120
5	$\{70\,,\, 8.00\}$	70

The last internal offer at † that gets cleared is $\beta_3^{\dagger}$. A 1 MW increase in demand will be met by clearing an additional 1 MW from the offer $\beta_3^{\dagger} = \{120\,,\, 10\}$, hence the marginal price of electricity at † is $\chi^{\dagger} = 10$ *\$/MWh*. Table 4.15 lists the internal offers at ‡ and the cleared power from each offer in order to satisfy $230\,MW$. The last internal offer at ‡ that was cleared was $\beta_2^{\ddagger}$. A 1 MW increase in demand will be met by clearing an additional 1 MW from the offer $\beta_2^{\ddagger} = \{130\,,\, 9\}$, hence the marginal price of electricity at † is $\chi^{\ddagger} = 9$ *\$/MWh*.

Following the discussion in Section 3.4, only the interface offers with price differences that are less than or equal to $\chi_{\dagger}$ - $\chi_{\ddagger}$ are cleared. Hence, $\beta_1^{\ddagger\rightarrow\dagger} = \{20\,,\,0.75\}$ and $\beta_2^{\ddagger\rightarrow\dagger} = \{30\,,\,1.00\}$ are cleared, and the interface offers $\beta_3^{\ddagger\rightarrow\dagger} = \{40\,,\,1.25\}$ and $\beta_4^{\ddagger\rightarrow\dagger} = \{30\,,\,1.50\}$ are rejected. Finally, the total payments are obtained as $\chi_{\dagger} \times \sum_{i=1}^{I} a_i^{\dagger} + \chi_{\ddagger} \times \sum_{j=1}^{J} a_j^{\ddagger}$=(10 $\times$ 220) + (9 $\times$ 230)=\$ 4270.

Table 4.15: Clearance of the original internal offers at ‡ and ‡ by CTS

i	$\beta_j^{\ddagger} = \{p_j^{\ddagger}\,,\,\mu_j^{\ddagger}\}$	$a_j^{\ddagger*}$
1	$\{150\,,\,8\}$	150
2	$\{130\,,\,9\}$	80
3	$\{100\,,\,10\}$	0
4	$\{130\,,\,10.5\}$	0
5	$\{120\,,\,12\}$	0
6	$\{100\,,\,13\}$	0

4.3 Illustrative Example 2

In this section, we use the Data Set 2 presented in Section 4.1 to demonstrate the steps of the CTS as explained in Chapter 3. Similar to Section 4.1, the two interconnected IGOs need to collect the required data/information, which serve as inputs to the models that get accommodated in the CTS framework. The internal offers at † that are listed in Table 4.5 and the intra-hour demand at † serve as inputs to the IGO^M internal market model in the economic layer, and the internal offers at ‡ that are listed in Table 4.6 and the intra-hour demand at ‡ serve as inputs to the IGO^F internal market model in the economic layer. The † to ‡ interface offers in Table 4.7 and the ‡ to † interface offers in Table 4.8 are provided as inputs to the CTS model in the economic layer.

The data/information need to be passed from the respective models to the CTS model for the execution of CTS. The IGO^M (IGO^F) internal market model passes the internal offers at † (‡) and the intra-hour demand at † (‡) to the CTS model. The required data/information are collected in the CTS model.

<u>CTS Step 1: Determination of the interface exchange direction</u>

The first step of CTS is the determination of the interface exchange direction. Similar to Section 4.2, the CTS model solves an ED problem for each IGO that does not take into account the interface exchange. The optimization variable of the ED problem for IGO^M is $\acute{a}_i^\dagger$, which represents the cleared real power in MW from the i^{th} submitted internal offer at † for the ED problem described in this step. The ED problem solved for IGO^M, formulated in (4.1), is reprised here:

$$\begin{aligned} &\min_{\acute{a}_i^\dagger} \quad \sum_{i=1}^{I} (\acute{a}_i^\dagger)\,(\mu_i^\dagger) \\ s.t. \ &\sum_{i=1}^{I} \acute{a}_i^\dagger \ - \ d^\dagger \ = \ 0 \quad \longleftrightarrow \ \acute{\chi}^\dagger \\ &\acute{a}_i^\dagger \ - \ p_i^\dagger \leq 0 \quad i \ = \ 1, 2, ...I \end{aligned} \tag{4.5}$$

where $d^\dagger = 150\ MW$, and $p_i^\dagger$ and $\mu_i^\dagger$ of the respective internal offer i at † are listed in Table 4.5, for $i = 1, .., I$ and $I = 4$. The objective of the optimization problem is to meet the demand $d^\dagger = 150\ MW$ in the cheapest way from the internal offers at † that are listed in Table 4.5. By observation, we can solve (4.5) by clearing the internal offers at Table 4.5 in the order of lowest to highest offer prices. Table 4.16 lists the cleared internal offer amounts to meet the demand $d^\dagger = 150\ MW$.

Table 4.16: Clearance of the internal offers at † for CTS Step 1

i	$\beta_i^\dagger = \{p_i^\dagger\,,\,\mu_i^\dagger\}$	$\acute{a}_i^{\dagger *}$
4	$\{60\,,\,8.00\}$	60
3	$\{200\,,\,9.00\}$	90
2	$\{140\,,\,10.00\}$	0
1	$\{130\,,\,11.00\}$	0

The last internal offer at † that was cleared was $\beta_2^\dagger$. A 1 MW increase in demand will be met by clearing an additional 1 MW from the offer $\beta_2^\dagger = \{200\,,\,9\}$, hence the marginal price of electricity at † for the ED problem defined in (4.5) is $\acute{\chi}^\dagger = 9$ $\$/MWh$.

While the CTS model solves the ED problem (4.5) for IGO^M, it simultaneously solves the exact same problem for IGO^F. The optimization variable of the ED problem for IGO^F is $\acute{a}_j^\ddagger$, which represents the cleared real power in MW from the j^{th} submitted internal offer for the ED problem described in this step. The ED

problem is formulated as follows:

$$
\begin{aligned}
& \min_{\acute{a}_j^{\ddagger}} \quad \sum_{j=1}^{J} (\acute{a}_j^{\ddagger})\,(\mu_j^{\ddagger}) \\
& s.t. \; \sum_{j=1}^{J} \acute{a}_j^{\ddagger} \; - \; d^{\ddagger} \; = \; 0 \quad \longleftrightarrow \; \acute{\chi}^{\dagger} \\
& \acute{a}_j^{\ddagger} \; - \; p_j^{\ddagger} \leq \; 0 \quad j \; = \; 1, 2, \ldots J
\end{aligned}
\tag{4.6}
$$

where $d^{\ddagger} = 270\ MW$, and $p_j^{\ddagger}$ and $\mu_j^{\ddagger}$ of the respective internal offer j at $\ddagger$ are listed in Table 4.6, for $j = 1, .., J$ and $J = 5$.

The objective of the optimization problem is to meet the demand $d^{\ddagger} = 270\ MW$ in the cheapest way from the internal offers at $\ddagger$ that are listed in Table 4.6. By observation, we can solve (4.6) by clearing the internal offers at Table 4.6 in the order of lowest to highest offer prices. Table 4.17 lists the cleared internal offer amounts to meet the demand $d^{\ddagger} = 270\ MW$.

Table 4.17: Clearance of the internal offers at ‡ for CTS Step 1

j	$\beta_j^{\ddagger} = \{p_j^{\ddagger}\,,\, \mu_j^{\ddagger}\}$	$\acute{a}_j^{\ddagger *}$
1	$\{80\,,\,8\}$	80
2	$\{90\,,\,9\}$	90
3	$\{110\,,\,10\}$	100
4	$\{140\,,\,11\}$	0
5	$\{100\,,\,12\}$	0

The last internal offer at ‡ that was cleared was $\beta_3^{\ddagger}$. A 1 MW increase in demand will be met by clearing an additional MW from the offer $\beta_3^{\ddagger} = \{110\,,\,10\}$, hence the marginal price of electricity at ‡ for the ED problem defined in (4.6) is $\acute{\chi}^{\ddagger} = 10$ $\$/MWh$.

The CTS procedure allows the interface exchange to be only from the bus with the lower marginal price to the bus with the higher marginal price. Since $\acute{\chi}^{\ddagger} = 10$ $\$/MWh$ and $\acute{\chi}^{\dagger} = 9$ $\$/MWh$, the interface exchange direction is from the bus † of IGO^M to the bus ‡ of IGO^F.

CTS Step 2: Modification of the internal offers at †

The interface exchange direction that was determined in the previous step of CTS designates how the internal offers at the bus † are modified by the interface offers. Since the interface exchange is from the bus † of IGO^M to the bus ‡ of IGO^F, the † to ‡ interface offers are used to modify the internal offers at †, and the ‡ to † interface offers are not considered. Following our discussion in Section 3.3, we arrange the internal offers at † in descending order of their offer prices in Table 4.18. Further, we arrange the † to ‡ interface offers in the ascending order of their price differences in Table 4.19.

Table 4.18: Internal offers at † arranged in descending order of offer prices

i	$\beta_i^\dagger = \{p_i^\dagger\,,\,\mu_i^\dagger\}$
1	$\{130\,,\,11.00\}$
2	$\{140\,,\,10.00\}$
3	$\{200\,,\,9.00\}$
4	$\{60\,,\,8.00\}$

Table 4.19: † to ‡ interface offers arranged in ascending order of offered price differences

n	$\beta_n^{\dagger\to\ddagger} = \{p_n^{\dagger\to\ddagger}\,,\,\mu_n^{\dagger\to\ddagger}\}$
1	$\{30\,,\,0.50\}$
2	$\{50\,,\,0.75\}$
3	$\{40\,,\,1.25\}$
4	$\{30\,,\,1.50\}$
5	$\{40\,,\,1.75\}$

We follow the substeps provided in Section 3.3 and modify the internal offers as follows:

$\beta_1^{\dagger} = \{130\,,\,11.00\}$ is modified by $\beta_1^{\dagger\to\ddagger} = \{30\,,\,0.50\}$ to create $\tilde{\beta}_1^{\dagger} = \{30, 11.50\}$ and $\tilde{\beta}_5^{\dagger} = \{100, 11.00\}$.

$\beta_2^{\dagger} = \{140\,,\,10.00\}$ is modified by $\beta_2^{\dagger\to\ddagger} = \{50\,,\,0.75\}$ to create $\tilde{\beta}_2^{\dagger} = \{50, 10.75\}$ and $\tilde{\beta}_6^{\dagger} = \{90, 10.00\}$.

$\beta_3^{\dagger} = \{200\,,\,9.00\}$ is modified by $\beta_3^{\ddagger\to\dagger} = \{40\,,\,1.25\}$ to create $\tilde{\beta}_3^{\dagger} = \{40, 10.25\}$ and $\tilde{\beta}_7^{\dagger} = \{160, 9.00\}$.

$\beta_4^{\dagger} = \{60\,,\,8.00\}$ is modified by $\beta_4^{\ddagger\to\dagger} = \{30\,,\,1.50\}$ to create $\tilde{\beta}_4^{\dagger} = \{30, 9.50\}$ and $\tilde{\beta}_8^{\dagger} = \{30, 8.00\}$.

CTS Step 3: Determination of the interface exchange amount

After the modified internal offers at †, $\{\tilde{\beta}^{\dagger}\}$, are obtained in the previous step, we follow the discussion in Section 3.4 and determine the interface exchange amount as a by-product of the solution of yet another ED problem for the hypothetical aggregation of the two interconnected IGOs as a single system based on the modified internal offers at † $\{\tilde{\beta}^{\dagger}\}$, the internal offers ‡ $\{\beta^{\ddagger}\}$, and the combined forecasted intra-hour demands at † and ‡, $d^{\dagger} + d^{\ddagger}$.

The ED problem for the fictitiously aggregated system is formulated in (4.7).

$$
\begin{aligned}
&\min_{\tilde{a}_{\tilde{i}}^{\dagger},\, a_j^{\ddagger}} \quad \sum_{\tilde{i}=1}^{\tilde{I}} (\tilde{a}_{\tilde{i}}^{\dagger})\,(\tilde{\mu}_{\tilde{i}}^{\dagger}) \;+\; \sum_{j=1}^{J} (a_j^{\ddagger})\,(\mu_j^{\ddagger}) \\
&s.t. \quad \sum_{\tilde{i}=1}^{\tilde{I}} \tilde{a}_{\tilde{i}} + \sum_{j=1}^{J} a_j \;-\; (\bar{d}^{\dagger} + \bar{d}^{\ddagger}) \;=\; 0 \quad \longleftrightarrow \quad \tilde{\chi} \\
&\qquad \tilde{a}_{\tilde{i}} \;-\; \tilde{p}_{\tilde{i}} \leq 0 \quad \tilde{i} \;=\; 1, 2, ...\tilde{I} \\
&\qquad a_j \;-\; p_j \leq 0 \quad j \;=\; 1, 2, ...J
\end{aligned}
\tag{4.7}
$$

The optimal decision variables ${\tilde{a}_{\tilde{i}}^{\dagger}}^{*}$ $\tilde{i} \;=\; 1, 2, ...\tilde{I}$ and ${a_j^{\ddagger}}^{*}$ $j \;=\; 1, 2, ...J$ indicate the MW of power cleared from the internal offers $\tilde{i} \;=\; 1, 2, ...\tilde{I}$ and $j \;=\; 1, 2, ...J$. In order to illustrate the solution of the optimization problem (4.7), we form a table that contains the modified internal offers at † $\{\tilde{\beta}^{\dagger}\}$ and the internal offers at ‡ $\{\beta^{\ddagger}\}$ arranged in ascending order of their offer prices. Given that $d^{\dagger} = 150\,MW$ and $d^{\ddagger} = 270\,MW$, we select the offers such that $d^{\dagger} + d^{\ddagger} = 420\,MW$ is met from the cheapest $420\,MW$ offers. Table 4.20 lists the cleared modified internal offer amounts at † and the cleared internal offer amounts at ‡ to meet the demand $d^{\dagger} + d^{\ddagger} = 420\,MW$.

The total power sold by the offerers at † can be represented by $\sum_{\tilde{i}=1}^{\tilde{I}} \tilde{a}_{\tilde{i}}^{\dagger^*}$. From Table 20, the total power sold by the internal offerers at † is $235\,MW$. Similarly, the total power sold by the internal offerers at ‡ can be represented as $\sum_{j=1}^{J} a_j^{\ddagger^*}$. Also from Table 20, the total power sold by the internal offerers at ‡ is $185\,MW$.

Plugging the values $d^{\dagger} = 150\,MW$, $d^{\ddagger} = 270\,MW$, $\sum_{\tilde{i}=1}^{\tilde{I}} \tilde{a}_{\tilde{i}}^{\dagger^*} = 235\,MW$, and $\sum_{j=1}^{J} a_j^{\ddagger^*} = 185\,MW$ in (4.8) as explained in Section 3.3,

$$\begin{aligned} \sum_{j=1}^{J} a_j^{\ddagger^*} &= d^{\ddagger} + f_{\ell} \\ \sum_{\tilde{i}=1}^{\tilde{I}} \tilde{a}_{\tilde{i}}^{\dagger^*} + f_{\ell} &= d^{\dagger} \end{aligned} \tag{4.8}$$

we obtain $f_{\ell} = -85\,MW$.

Table 4.20: Clearance of the modified internal offers at † and the internal offers at ‡ for the solution of (4.3)

$\beta = \{p\,,\, \mu\}$	a
$\beta_1^{\ddagger} = \{80\,,\, 8.00\}$	$a_1^{\ddagger} = 80$
$\tilde{\beta}_8^{\dagger} = \{30\,,\, 8.00\}$	$\tilde{a}_8^{\dagger} = 30$
$\beta_2^{\ddagger} = \{90\,,\, 9.00\}$	$a_2^{\ddagger} = 90$
$\tilde{\beta}_7^{\dagger} = \{160\,,\, 9.00\}$	$\tilde{a}_7^{\dagger} = 160$
$\tilde{\beta}_4^{\dagger} = \{30\,,\, 9.50\}$	$\tilde{a}_4^{\dagger} = 30$
$\beta_3^{\ddagger} = \{110\,,\, 10.00\}$	$a_3^{\ddagger} = 15$
$\tilde{\beta}_6^{\dagger} = \{90\,,\, 10.00\}$	$\tilde{a}_6^{\dagger} = 15$
$\tilde{\beta}_3^{\dagger} = \{40\,,\, 10.25\}$	$\tilde{a}_3^{\dagger} = 0$
$\tilde{\beta}_2^{\dagger} = \{50\,,\, 10.75\}$	$\tilde{a}_2^{\dagger} = 0$
$\beta_4^{\ddagger} = \{140\,,\, 11.00\}$	$a_4^{\ddagger} = 0$
$\tilde{\beta}_5^{\dagger} = \{100\,,\, 11.00\}$	$\tilde{a}_5^{\dagger} = 0$
$\tilde{\beta}_1^{\dagger} = \{30\,,\, 11.50\}$	$\tilde{a}_1^{\dagger} = 0$
$\beta_5^{\ddagger} = \{100\,,\, 12\}$	$a_5^{\ddagger} = 0$

CTS Step 4: Clearance of the internal offers and the interface offers

In this step, the CTS model clears the original internal offers at † so that $\sum_{\tilde{i}=1}^{\tilde{I}} \tilde{a}_{\tilde{i}}^{\dagger *} = 235\,MW$ is met in the most economical way. Table 4.21 provides the internal offers at † and the cleared power amount from each offer in order to satisfy $235\,MW$.

Table 4.21: Clearance of the original internal offers at † and ‡ by CTS

i	$\beta_i^{\dagger} = \{p_i^{\dagger}\,,\, \mu_i^{\dagger}\}$	$\acute{a}_i^{\dagger}$
4	$\{60\,,\, 8.00\}$	60
3	$\{200\,,\, 9.00\}$	175
2	$\{140\,,\, 10.00\}$	0
1	$\{130\,,\, 11.00\}$	0

The last internal offer at † that gets cleared is $\beta_3^{\dagger}$. A 1 MW increase in demand will be met by clearing an additional 1 MW from the offer $\beta_3^{\dagger} = \{200\,,\, 9.00\}$, hence the marginal price of electricity at † is $\chi^{\dagger} = 9$ $\$/MWh$.

In addition, Table 4.22 provides the internal offers at ‡ and the cleared power amount from each offer in order to satisfy $185\,MW$. The last internal offer at ‡ that is cleared in Table 4.22 is $\beta_3^{\ddagger}$. A 1 MW increase in demand will be met by clearing an additional 1 MW from the offer $\beta_3^{\ddagger} = \{110\,,\, 10\}$, hence the marginal price of electricity at † is $\chi^{\ddagger} = 10$ $\$/MWh$.

Following the discussion in Section 3.4, only the interface offers with price differences less than or equal to $\chi_{\ddagger}$ - $\chi_{\dagger}$ $= 1$ $\$/MWh$ are cleared. Hence, $\beta_1^{\dagger\rightarrow\ddagger} = \{30\,,\,0.50\}$ and $\beta_2^{\dagger\rightarrow\ddagger} = \{50\,,\,0.75\}$ are cleared, and the interface offers $\beta_3^{\dagger\rightarrow\ddagger} = \{40\,,\,1.25\}$, $\beta_4^{\dagger\rightarrow\ddagger} = \{30\,,\,1.50\}$, and $\beta_4^{\dagger\rightarrow\ddagger} = \{40\,,\,1.75\}$ are rejected. Finally, the total payments are obtained as $(\chi_{\dagger} \times \sum_{i=1}^{I} a_i^{\dagger}) + (\chi_{\ddagger} \times \sum_{j=1}^{J} a_j^{\ddagger})$=$(9 \times 235) + (10 \times 185)$= $\$3965$.

Table 4.22: Clearance of the original internal offers at ‡ and ‡ by CTS

j	$\beta_j^{\ddagger} = \{p_j^{\ddagger}\,,\,\mu_j^{\ddagger}\}$	$\acute{a}_j^{\dagger}$
1	$\{80\,,\,8\}$	80
2	$\{90\,,\,9\}$	90
3	$\{110\,,\,10\}$	15
4	$\{140\,,\,11\}$	0
5	$\{100\,,\,12\}$	0

4.4 Sensitivity Analyses

In the previous two sections, we have illustrated the execution of CTS with two different data sets. In this section, we conduct various sensitivity analyses, and evaluate the dependence of the interface flow and the total payments on a change in submitted internal or interface offers.

Change in the internal offer $\beta_5^{\dagger}$ of Data Set 1

In order to illustrate the dependence of the interface flow and the total payments on a change in a submitted internal offer, we change a submitted internal offer in Data Set 1 and repeat Illustrative Example 1. We change the submitted internal offer $\beta_5^{\dagger} = \{70\,,\, 8.00\}$ of Data Set 1 to $\beta_5^{\dagger} = \{120\,,\, 8.00\}$, and repeat the execution of CTS with the new set of submitted internal offers. The first step of CTS, as demonstrated in Section 4.2, is the determination of the interface exchange direction. In order to do that, each side solved an ED problem without considering the interface exchange, and obtained the marginal price of electricity. We have seen in Illustrative Example 1 that $\acute{\chi}^{\ddagger}$ was obtained as 10 $\$/MWh$. The change in the submitted internal offer $\beta_5^{\dagger}$ of Data Set 1 to $\beta_5^{\dagger} = \{120\,,\, 8.00\}$ may result in a change in $\acute{\chi}^{\dagger}$. In order to observe that, we solve (4.1) again with the new set of submitted internal offers. We clear the new set of internal offers at † in the order of lowest to highest offer prices so as to meet the demand $d^{\dagger} = 230\,MW$. Table 4.23 lists the clearance of the new set of internal offers at † for the ED problem defined in (4.1).

Table 4.23: Clearance of the new set of internal offers at † for CTS Step 1 for $\beta_5^{\dagger} = \{70\,,\,8.00\}$

i	$\beta_i^{\dagger} = \{p_i^{\dagger}\,,\,\mu_i^{\dagger}\}$	$\acute{a}_i^{\dagger *}$
1	$\{170\,,\,12.00\}$	0
2	$\{150\,,\,11.00\}$	0
3	$\{120\,,\,10.00\}$	0
4	$\{120\,,\,9.00\}$	110
5	$\{120\,,\,8.00\}$	120

The last internal offer at † that was cleared is now $\beta_4^{\dagger}$. A 1 MW increase in demand will be met by clearing an additional 1 MW from the offer $\beta_4^{\dagger} = \{120\,,\,9\}$, hence the marginal price of electricity at † is now $\acute{\chi}^{\dagger} = 9$ $\$/MWh$ whereas it used to be 10 $\$/MWh$. Since $\acute{\chi}^{\ddagger} = 9$ $\$/MWh$ as well, the marginal prices of electricity on both sides of the interface are the same, and there will not be an interface exchange in CTS, whereas f_{ℓ} was $10\,MW$ before the change of internal offer $\beta_5^{\dagger}$. Hence, no interface offers will be accepted. The total payments are now obtained as $(\chi_{\dagger} \times \sum_{i=1}^{I} a_i^{\dagger *}) + (\chi_{\ddagger} \times \sum_{j=1}^{J} a_j^{\ddagger *})$=(9 × 230) + (9 × 220)=\$4050.

Change in the interface offer $\beta_4^{\ddagger\rightarrow\dagger}$ of Data Set 1

We also observe the dependence of the interface flow and the total payments on a change in the submitted interface offers. We change the submitted interface offer $\beta_4^{\ddagger\rightarrow\dagger} = \{30\,,\, 1.50\}$ to $\beta_4^{\ddagger\rightarrow\dagger} = \{50\,,\, 1.50\}$, which indicates that the interface offerer is willing to transact more power for the same price difference between the two busses. When the interface offer is modified as such, the modification on the internal offer at † will be different, and $\beta_4^{\dagger} = \{120\,,\, 9.00\}$ will instead be modified by $\beta_4^{\ddagger\rightarrow\dagger} = \{50\,,\, 1.50\}$ to create $\tilde{\beta}_4^{\dagger} = \{50, 7.50\}$ and $\tilde{\beta}_9^{\dagger} = \{70, 9.00\}$. Such a change in the modified internal offers will influence CTS Step 3, as (4.3) will be solved with the new set of modified internal offers. Table 4.24 indicates the cleared modified internal offer amounts at † created by $\beta_4^{\ddagger\rightarrow\dagger} = \{50\,,\, 1.50\}$ and the cleared internal offer amounts at ‡ to meet the demand $d^{\dagger} + d^{\ddagger} = 450\,MW$.

The total power sold by the offerers at † can be represented by $\sum_{\tilde{i}=1}^{\tilde{I}} \tilde{a}_{\tilde{i}}^{\dagger}$. From Table 4.24, the total power sold by the internal offerers at † is $230\,MW$. Similarly, the total power sold by the internal offerers at ‡ can be represented by $\sum_{j=1}^{J} a_j^{\ddagger}$. Also from Table 4.24, the total power sold by the internal offerers at ‡ is $220\,MW$. Plugging the values $d^{\dagger} = 230\,MW$, $d^{\ddagger} = 220\,MW$, $\sum_{\tilde{i}=1}^{\tilde{I}} \tilde{a}_{\tilde{i}}^{\dagger} = 220\,MW$, and $\sum_{j=1}^{J} a_j^{\ddagger} = 230\,MW$

in (4.9)as explained in Section 3.3

$$\begin{aligned}\sum_{j=1}^{J} a_j^{\ddagger *} &= d^{\ddagger} + f_{\ell} \\ \sum_{\tilde{i}=1}^{\tilde{I}} \tilde{a}_{\tilde{i}}^{\dagger *} + f_{\ell} &= d^{\dagger}\end{aligned} \tag{4.9}$$

we obtain $f_\ell = 0\,MW$, where f_ℓ used to be $10\,MW$ before the change of the interface offer.

When CTS Step 4 is executed, the total power sold by the internal offerers at † $230\,MW$ will be cleared from the original internal offers at † in the order of lowest to highest offer prices. Table 4.25 lists the internal offers at † and the cleared power amount from each offer in order to satisfy $230\,MW$ in the most economical way. From Table 4.25, the last internal offer at † that gets cleared is $\beta_3^{\dagger}$. A 1 MW increase in demand will be met by clearing an additional 1 MW from the offer $\beta_3^{\dagger} = \{120\,,\,10\}$, hence the marginal price of electricity at † is $\chi^{\dagger} = 10$ $\$/MWh$.

Table 4.26 lists the internal offers at ‡ and the cleared power from each offer in order to satisfy $220\,MW$. From Table 4.26, the last internal offer at ‡ that was cleared was $\beta_2^{\ddagger}$. A 1 MW increase in demand will be met by clearing an additional 1 MW from the offer $\beta_2^{\ddagger} = \{130\,,\, 9\}$, hence the marginal price of electricity at † is $\chi^{\ddagger} = 9$ $\$/MWh$.

Following the discussion in Section 3.4, only the interface offers with price differences less than or equal to $\chi_{\dagger}$ - $\chi_{\ddagger}$ are cleared. Since the price difference between the two busses remained the same after the change in $\beta_4^{\ddagger\rightarrow\dagger}$, the same interface offers are cleared. Hence, $\beta_1^{\ddagger\rightarrow\dagger} = \{20\,,\, 0.75\}$ and $\beta_2^{\ddagger\rightarrow\dagger} = \{30\,,\, 1.00\}$ are cleared, and the interface offers $\beta_3^{\ddagger\rightarrow\dagger} = \{40\,,\, 1.25\}$ and $\beta_4^{\ddagger\rightarrow\dagger} = \{30\,,\, 1.50\}$ are rejected. Finally, the total payments are obtained as $\chi_{\dagger} \times \sum_{i=1}^{I} a_i^{\dagger} + \chi_{\ddagger} \times \sum_{j=1}^{J} a_j^{\ddagger}$=(10 × 230) + (9 × 220)=\$ 4280. Before the change in the interface offer $\beta_4^{\ddagger\rightarrow\dagger}$, the total payments were obtained as $\$$ 4270, hence when the submitted interface offer $\beta_4^{\ddagger\rightarrow\dagger} = \{30\,,\, 1.50\}$ is changed to $\beta_4^{\ddagger\rightarrow\dagger} = \{50\,,\, 1.50\}$, the total payments increased from $\$$ 4270 to $\$$ 4280.

Table 4.24: Clearance of the modified internal offers at † and the internal offers at ‡ for the solution of (4.3) for $\beta_4^{\ddagger \to \dagger} = \{50\,,\,1.50\}$

$\beta = \{p\,,\,\mu\}$	a^*
$\tilde{\beta}_4^{\dagger} = \{50\,,\,7.5\}$	$\tilde{a}_4^{\dagger} = 50$
$\tilde{\beta}_5^{\dagger} = \{70\,,\,8\}$	$\tilde{a}_5^{\dagger} = 70$
$\beta_1^{\ddagger} = \{150\,,\,8\}$	$a_1^{\ddagger} = 150$
$\tilde{\beta}_3^{\dagger} = \{40\,,\,8.75\}$	$\tilde{a}_3^{\dagger} = 40$
$\tilde{\beta}_9^{\dagger} = \{70\,,\,9\}$	$\tilde{a}_9^{\dagger} = 70$
$\beta_2^{\ddagger} = \{130\,,\,9\}$	$a_2^{\ddagger} = 70$
$\tilde{\beta}_2^{\dagger} = \{30\,,\,10\}$	$\tilde{a}_2^{\dagger} = 0$
$\tilde{\beta}_8^{\dagger} = \{80\,,\,10\}$	$\tilde{a}_8^{\dagger} = 0$
$\beta_3^{\ddagger} = \{100\,,\,10\}$	$a_3^{\ddagger} = 0$
$\beta_4^{\ddagger} = \{130\,,\,10.5\}$	$a_4^{\ddagger} = 0$
$\tilde{\beta}_7^{\dagger} = \{120\,,\,11\}$	$\tilde{a}_7^{\dagger} = 0$
$\tilde{\beta}_1^{\dagger} = \{20\,,\,11.25\}$	$\tilde{a}_1^{\dagger} = 0$
$\tilde{\beta}_6^{\dagger} = \{150\,,\,12\}$	$\tilde{a}_6^{\dagger} = 0$
$\beta_5^{\ddagger} = \{120\,,\,12\}$	$a_5^{\ddagger} = 0$
$\beta_6^{\ddagger} = \{100\,,\,13\}$	$a_6^{\ddagger} = 0$

Table 4.25: Clearance of the original internal offers at † by CTS for $\beta_4^{\ddagger \rightarrow \dagger} = \{50\,,\,1.50\}$

i	$\beta_i^{\dagger} = \{p_i^{\dagger}\,,\,\mu_i^{\dagger}\}$	$a_i^{\dagger *}$
1	$\{170\,,\,12.00\}$	0
2	$\{150\,,\,11.00\}$	0
3	$\{120\,,\,10.00\}$	40
4	$\{120\,,\,9.00\}$	120
5	$\{70\,,\,8.00\}$	70

Table 4.26: Clearance of the original internal offers at ‡ by CTS for $\beta_4^{\ddagger \rightarrow \dagger} = \{50\,,\,1.50\}$

i	$\beta_j^{\ddagger} = \{p_j^{\ddagger}\,,\,\mu_j^{\ddagger}\}$	$a_j^{\ddagger *}$
1	$\{150\,,\,8\}$	150
2	$\{130\,,\,9\}$	70
3	$\{100\,,\,10\}$	0
4	$\{130\,,\,10.5\}$	0
5	$\{120\,,\,12\}$	0
6	$\{100\,,\,13\}$	0

4.5 Concluding Remarks and Summary

In this chapter, we have illustrated the execution of CTS, and run a sensitivity analysis to observe the dependence of the interface exchange and the total payments on a change in internal or interface offers. We have illustrated the execution of CTS with two data sets. In Section 4.2, we have observed that, without any use of the interface, the total payments were calculated as $\$4280$. When the coordination procedure CTS was used for the interchange exchange determination, we obtained the interface exchange as $f_\ell = 10\,MW$ and the total payments as $\$4270$, which showed that CTS was able to decrease the total payments. We have also observed the sensitivity of the internal offer and the interface offer to the interface exchange and the total payments.

We have observed that an increase in the offered power amount of an internal offer at † decreased the need to transact power from ‡, hence the interface exchange decreased. Furthermore, such a change also decreased the total payments obtained via CTS from $\$4270$ to $\$4050$.

CHAPTER 5
TOWARD SHORTHER COORDINATION PERIODS

The discussion of CTS in Chapter 3 showed that a key consideration in the execution of CTS is time. Since CTS includes steps that must be jointly executed by the two IGOs, it is required to have a thorough description of the instants by which each step must be executed, as well as the instants by which the necessary data/information must be shared, so that the two IGOs can coordinate the execution of CTS. Furthermore, CTS relies on the demand forecast and the submitted internal and interface offers to determine the interchange evaluation. Based on forecasting experience, we know that the variation of a forecast error increases with time. As such, it is desired to reduce the duration of the coordination period, so that the CTS yields an interface flow that is close to the interface flow that would have been obtained, had the actual values been used. The CTS determines the interface flow for the coordination period $[k]\big|_h$, and the determined interface flow remains constant during the entire coordination period. The real-time markets of the two interconnected IGOs, on the other hand, may have shorter durations. Our analysis of CTS in Chapter 3 made it clear that the interface flow determined for the coordination period $[k]\big|_h$ is from the bus with the lower marginal price to the bus with the higher marginal price—which is the intuitive interface flow direction. However, if the duration of the real-time markets is shorter than the duration of the coordination period $[k]\big|_h$, then the determined interface flow may be from the bus with the higher marginal price to the bus with the lower marginal price for certain real-time market periods—which is a counter-intuitive interface flow direction. Hence, we are interested in observing the influence of the duration of the coordination periods on the number of real-time market periods during which we observe counter-intuitive interface flows.

In order to perform the analysis on the time issues of CTS, we focus on the coordination period $[k]\big|_h$, and reprise our time convention discussed in Section 2.1. For a given day, we express all time elements in minutes (m) and a time element τ denotes the cumulative minutes elapsed from the start of the day, at 0 min. The term h is the index for the hourly periods of a day such that $h = 1, 2, ..., 24$. We define the hourly period as $\mathcal{T}_h \triangleq \{\tau : (h-1)(60) < \tau \leq (h)(60)\}$.

We consider K intra-hourly subperiods of equal duration for each hour h, and introduce the index k for the intra-hourly subperiods such that $k = 1, 2, ..., K$. We denote the duration of each intra-hourly subperiod by ζ. We define the inta-hourly subperiod as $\mathcal{T}_{[k]}\big|_{h} \triangleq \{\tau : (h-1)(60) + (k-1)(\zeta) < \tau \leq (h-1)(60) + (k)(\zeta)\}$. The time frame of our analysis is illustrated in Figure 5.1.

In Section 5.1 we focus on the duration of the procedural steps on CTS, and discuss the instants at which each step must be completed. In Section 5.2, we examine the influence of decreasing coordination period durations on the performance of the demand forecasts. In Section 5.3, we utilize the real-time prices at the Sandy Pond bus, which is the $NYISO$'s proxy bus for ISO-NE, and the Roseton bus, which is the ISO-NE's proxy bus for $NYISO$, to analyze the influence of the coordination period durations on the number of real-time market periods at which we observe counter-intuitive interface flows. Finally, we provide the concluding remarks and the summary of the chapter in Section 5.4.

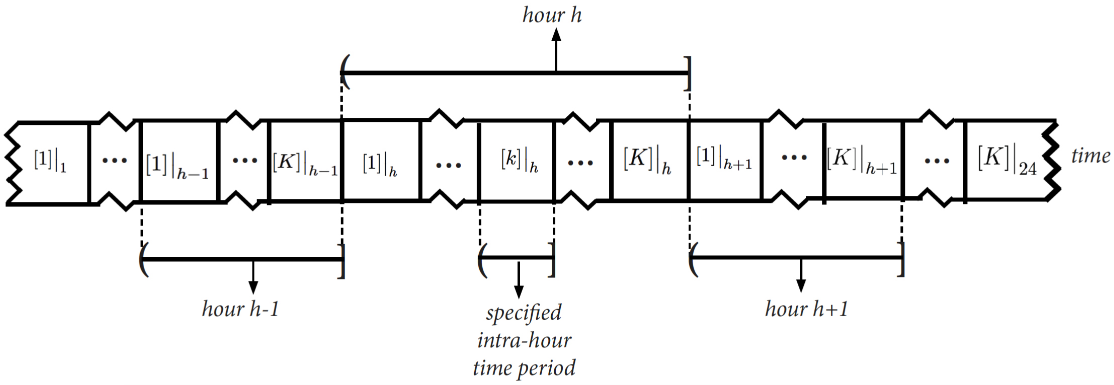

Figure 5.1: Time scale of the framework

5.1 Duration of the Procedural Steps of *CTS*

The discussion on CTS in Chapter 3 provided a detailed description of the steps that must be completed for the execution of CTS. In this section, we will specify the time instants by which these steps must be completed, as well as the factors that influence the duration of these steps.

Before the two interconnected IGOs can start CTS, they need to collect the necessary data/information, which are explained in detail in Section 3.1. We denote the time instant by which the necessary data/information must be collected by τ_p which depends on the internal mechanisms by which each IGO collects data/information from its system. The collected data/information include the intra-hour demand, internal offers, interface offers, and the data/information about the generators in that system. Each IGO may designate a different point in time as the deadline to collect these data/information from their individual system; however, they must jointly agree on τ_p as the time instant by which all the required data/information must be collected in order to start CTS.

The first step of CTS is the determination of the interface flow direction. The discussion in Section 3.2 shows that, in order to determine the interface flow direction, an ED problem is solved based on the internal offers and the intra-hour demands of each system without the utilization of the interface. The interface flow direction is determined to be from the bus with lower marginal price of electricity to the bus with higher marginal price of electricity. We denote the time instant by which the interface flow direction must be determined by τ_d. Time instant τ_d is predominantly influenced by the execution time of the software deployed to solve the ED problem in the two IGOs, and the software with the larger execution time determines τ_d. The execution time of the software may be influenced by the time it takes to input/output data to/from the software, the algorithm used by the software to solve the ED, and the number of the internal offers of the IGO.

In the second step of CTS, the internal offers at † are modified by the interface offers. The discussion in Section 3.3 explains the cases, as well as the substeps, that must be executed for each case to modify the internal offers at †. We denote the time instant by which the internal offers at † are modified by τ_m. Time instant τ_m depends on the execution time of the software that the CTS model of the framework deploys to modify the internal offers at †. The execution time of the software to modify the internal offers at † may be influenced by the time it takes to input/output data to/from the software, the number of the internal offers at †, and the number of interface offers.

Once the internal offers at † are modified, in Step 3, the CTS model solves an ED problem based on the modified internal offers at †, internal offers at ‡, and the aggregate intra-hour demands at † and ‡. The discussion in Section 3.4 shows that the solution of this ED problem yields the interface flow amount. We denote the time instant by which the ED problem in Step 3 must be solved and the interface flow amount must be determined by τ_t. Time instant τ_t is determined by the execution time of the software to solve the ED problem of this step. The execution time of the software may be influenced by the time it takes to input/output data to/from the software, the algorithm used by the CTS model to solve the ED defined in Step 3, and the number of the modified internal offers at † and the internal offers ‡.

In Step 4, the CTS model solves another ED problem for each system based on the original internal offers and the intra-hour demands at the bus of each system, and the interface flow that was determined in the previous step. These ED problems determine the cleared internal offers and the marginal price of electricity at each bus. Based on the difference between the marginal prices of electricity at the two busses, the interface offers are also cleared. We denote the time instant by which the internal offers and the interface offers are cleared by τ_a, which is determined by the execution time of the software used to solve the ED problems in this step. The execution time of the software may be influenced by the time it takes to input/output data to/from the software, the algorithm used to solve the ED problems in this step, the number of the internal offers at † and ‡, and the number of the interface offers.

The analysis in this section shows the significance of the consideration of time in the execution of CTS. In order for the two IGOs to perform CTS in a coordinated way, both IGOs must abide by the time instances by which each step must be completed, as an untimely completion of a step by one IGO may interrupt the performance of CTS. Furthermore, the completion time of each step depends on various factors, and in order to shorten the duration of the coordination period, the two IGOs must work on the factors that influence the duration of each step of CTS.

5.2 Influence of the Coordination Period Durations on the Demand Forecast Performance

CTS relies on the collected internal offers, interface offers and the forecasted intra-hour demands to determine the interchange evaluation. We have denoted the time instant by which the necessary data/information must be collected by τ_p. Some of the collected internal and interface offers that were submitted at τ_p may not be able to deliver and transact the power that they promised at $(h-1)(60)+k\zeta$, which is the beginning of the coordination period. Furthermore, the intra-hour demands that were forecasted at τ_p may differ from the actual demands observed at $(h-1)(60)+k\zeta$. Our forecasting experience shows that the forecast error increases with time. Hence, it is in our interest to work with shorter coordination periods, so that the interchange evaluation determined with the forecasted intra-hour demands will be closer to the value that would have been obtained, had the actual intra-hour demands been used for the determination of interchange evaluation.

In order to examine the influence of shorter coordination periods on the performance of the demand forecast, we consider two cases. In **Case 1**, we consider that each 5 *min* decrease in the duration of the coordination period increases the demand forecast performance by 3%. In **Case 2**, we consider that each 5 *min* decrease in the duration of the coordination period increases the demand forecast performance by 5%. In Figure 5.2, we plot the performance of the demand forecasts vs. different coordination periods for the two cases.

As expected, the plots in both cases show that as the coordination period gets shorter, the demand forecast performance increases. The demand forecast performance in **Case 1** ranges between 97.1% and 83.7% as the coordination period duration ranges between 5 *min* and 30 *min*. The demand forecast performance in **Case 2** ranges between 95.2% and 74.6% as the coordination period duration ranges between 5 *min* and 30 *min*. Since the duration of the real-time markets is generally 5 *min*, we select the duration of the coordination periods in our analysis so that it is an integer multiple of the duration of the real-time markets, *i.e.* 5 *min*. Figure 5.2 shows that in order to attain a 95% demand forecast performance, the coordination period durations in both cases must be 5 *min* or less. If it is desired to attain at least 90% demand forecast performance, then coordination period duration can be

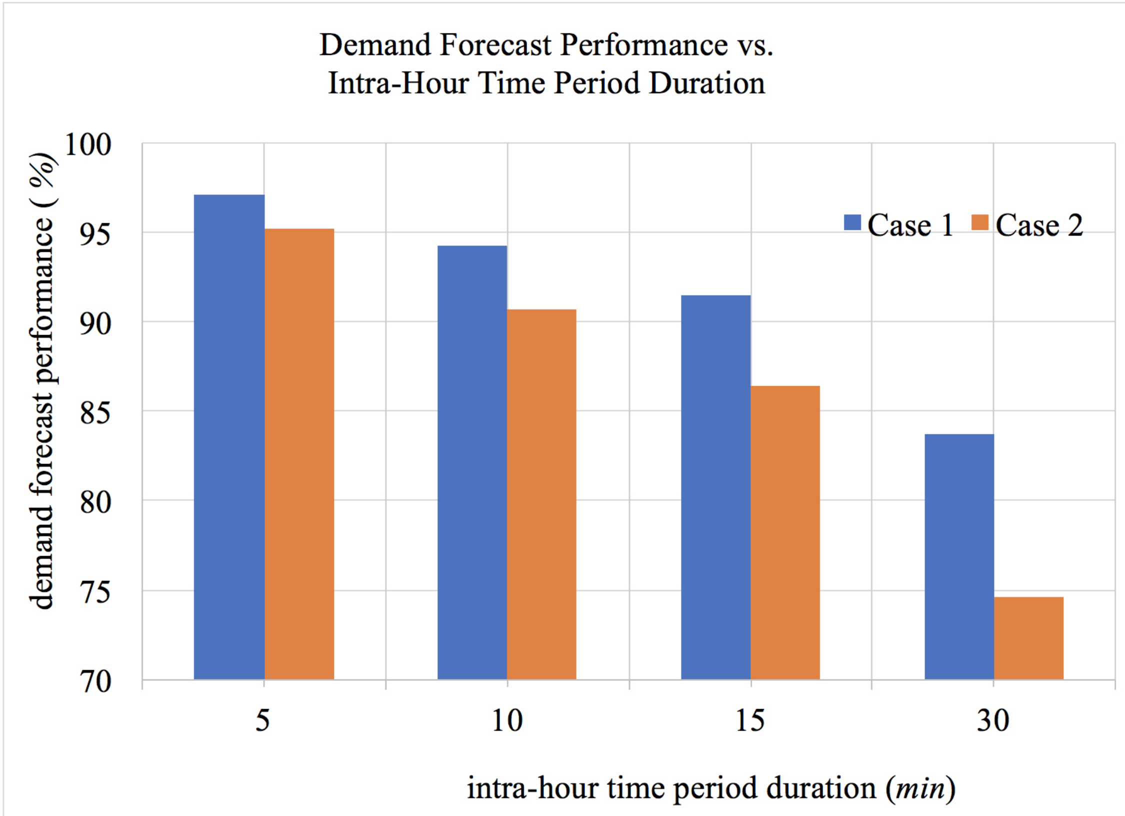

Figure 5.2: Demand forecast performance vs. coordination period duration

selected as 15 *min* for **Case 1**; however, the coordination period duration cannot be longer than 10 *min* for **Case 2**.

5.3 Influence of the Coordination Period Durations on the Counter-Intuitive Flows

The discussion of CTS in Chapter 3 has shown us that the interface flow is evaluated for the coordination period $[k]\big|_h$ that has a duration of ζ. The direction of the interface flow is determined so that it is from the bus at which the marginal price of electricity is lower to the bus at which the marginal price of electricity is higher—which is the intuitive interface flow direction—and the determined direction and the amount of the interface flow remain constant during the coordination period $[k]\big|_h$. However, this determination of the interface flow direction is based on the assumption that the marginal prices of electricity remain constant during the coordination period $[k]\big|_h$. We know that the real-time prices of electricity are determined in real-time markets, which in general have a duration of 5 min. If ζ is larger than the duration of the real-time market period, there may be real-time market periods at which the direction of the interface flow is counter-intuitive — from the bus at which the marginal price of electricity is higher to the bus at which the marginal price of electricity is lower. We anticipate that as ζ approaches the duration of the real-time market, we will observe fewer real-time market periods at which the interface flow direction is counter-intuitive, and that as ζ gets larger, we will observe more real-time market periods at which the interface flow direction is counter-intuitive.

In order to simulate the influence of the duration of coordination periods on the number of real-time market periods at which we observe counter-intuitive flows, we utilize the data on the real-time prices at the Sandy Pond bus of ISO-NE, and the Roseton bus of $NYISO$ for March 31, 2018. The real-time market duration is 5 min, and there are 288 real-time market periods in the selected day. We determine the direction of the interface flow for longer coordination periods by downsampling the real-time market price data. Namely, in order to observe the intra-hour market duration of 10 min, we downsample the real-time market price data by 2; in order to observe the intra-hour market duration of 15 min, we downsample the real-time market price data by 3; and in order to observe the intra-hour market duration of 30 min, we downsample the real-time market price data by 6. For each of these longer intra-hour market periods, we determine the direction of the interface flow based on the marginal prices at the busses of the samples. For instance, for a coordination period of 30 min, in order to determine the direction of the interface flow for the first 30 min of March 31, 2018, we compare the first real-time price data of the two busses. Then, we set the direction of the interface flow for the first 30 min so that it is from the bus with the lower marginal price to the bus with the higher marginal price of the first real-time price time data at the two busses. When we compare the first real-time market price data with the other 5 real-time market price data of the first coordination period, we may observe that the determined interface flow direction is from the bus with the higher marginal price for the real-time market to the bus with the lower marginal price for the real-time market—which is detrimental to the goal of the utilization of the interface.

In Figure 5.3, we provide the number of the real-time market periods at which the interface flow direction is counter-intuitive vs. the coordination period duration for March 30, 2018 between the Sandy Pond bus of $ISO\text{-}NE$, and the Roseton bus of $NYISO$.

We observe that as the coordination period duration increases, the number of real-time market periods at which the interface flow direction is counter-intuitive increases as well. Namely, when the coordination period duration is selected as 10 min, we observe 20 real-time market periods at which the direction of the interface flow is counter-intuitive. When the coordination period duration is selected as 15 min, we observe 32 real-time market periods at which the direction of the interface flow is counter-intuitive, and when the coordination period duration is selected as 30 min, we observe 41 real-time market periods at which the direction of the interface flow is counter-intuitive. Such an increase in the number of real-time market periods at which we observe counter-intuitive interface flow directions shows the significance of decreasing the coordination period durations. In order to utilize the interface flow so as to meet the demand with lower marginal price of electricity, it makes sense to decrease the coordination period durations. Hence, the ideal case for the coordination period duration would be to make it equal to the duration of the real-time markets. When the coordination period duration is equal to the real-time market duration, we can ensure that the interface flow directions are always intuitive at all real-time market periods under perfect information.

5.4 Concluding Remarks and Summary

In this chapter, we have examined the time issues of CTS. The interchange evaluation is determined jointly by the two IGOs and the two IGOs must decide on the time instances by which each step must be executed in order to perform CTS. We have stipulated the time instances by which each step of CTS must be executed, and identified the factors that influence the duration of the steps.

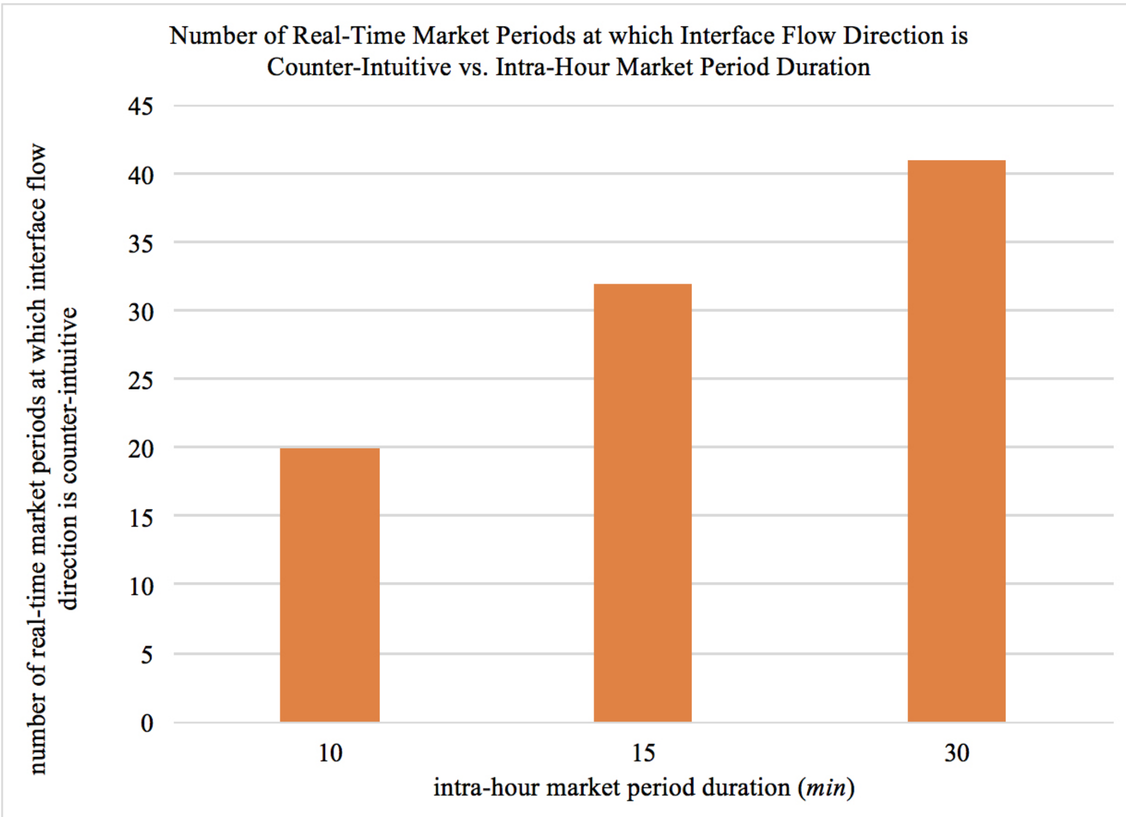

Figure 5.3: Number of the real-time market periods at which the interface flow direction is counter-intuitive vs. the coordination period duration

The interchange evaluation is determined using the demand forecasts and internal and interface offers, which are obtained a certain amount of time before CTS is executed. We anticipated that as the forecast gets closer in time, the performance of a forecast will increase. Hence, we have observed the influence of the duration of the coordination periods on the demand forecast performance for two cases, and evaluated the demand forecast performance for the coordination period durations of 5 *min*, 10 *min*, 15 *min*, and 30 *min*. We have observed that in both cases the demand forecast performance increases as the coordination period gets shorter. Further, in both cases, we have observed that a 5 *min* coordination period yields more than 95% of demand forecast performance.

We have also observed the influence of the duration of the coordination period on the number of real-time market periods at which counter-intuitive interface flows are observed. We have considered the real-time prices in the Sandy Pond and Roseton busses, and downsampled the real-time market data to determine the interface flow directions for longer coordination periods. We have observed that as the duration of the coordination period increases, the number of real-time market periods at which the interface flow is counter-intuitive increases as well. We conclude that the ideal would be to make the coordination period duration equal to the real-time market duration, which will theoretically ensure intuitive interface flow directions under perfect information at all real-time market periods.

CHAPTER 6

CONCLUSIONS

In this thesis, we focus on the analysis and the quantification of the performance of CTS. In order to accommodate all the required models and tools for the analysis and the quantification of a coordination scheme, we construct a two-layered framework. The two salient characteristics of the framework are generality and comprehensiveness. Since the framework is general, it allows the analysis and quantification of the performance of any coordination scheme, and the accommodation of any particular choice of model. The framework is comprehensive, as it takes into account all the physical and economic aspects of a coordination scheme.

We tailor the framework for CTS, and develop appropriate physical and economic models. We have identified the sharing of data/information as a key issue in the execution of a coordination scheme. We represent the sharing of data/information between the IGOs by the interactions between the models of the framework. In order to ensure that the data/information are shared only with intended parties, we design the interactions between the models according to the sharing specifications of data/information. We use the framework to discuss the procedural steps of CTS. We provide an analytical underpinning of the procedural steps of CTS and formulate the required tasks. The incorporation of interface offers in CTS enables the IGOs to harness market forces for the determination of interchange evaluation. Our discussion of the procedural steps of CTS showed that the modification of the internal offers by the interface offers acts as a barrier to the interface offerer that wishes to take advantage of the marginal electricity price difference between the two systems. These modifications also cause the total payments obtained via CTS to be higher than or equal to the total payments obtained via TLO.

We illustrate the performance of each procedural step of CTS using two data sets. Our results show that the total payment obtained via CTS is lower than the total payment that would have been obtained, had the two IGOs not utilized any interchange. We also conduct sensitivity analyses and evaluate the dependence of total payment and interface exchange amount on a change in internal or interface offers.

We observed that an increase in the offered transaction amount of an interface offer decreased the interface exchange amount. We also observed, that a change in an internal offer influenced the marginal price of electricity at a bus, and consequently influenced the interface exchange amount.

We assessed the elapsed time durations of the procedural steps of CTS and identified the limiting constraints, so as to work toward shorter coordination periods. We identified the limiting constraint on the time it takes to execute most procedural steps as the speed of the software in executing the respective tasks of the steps. We studied the dependence of the demand forecast performance on the coordination period duration, and simulated the demand forecast performance for different coordination period durations. We investigated the impact of coordination period durations on the number of real-time market periods in which the interface exchange is counter-intuitive. The duration of the real-time markets is generally shorter than the coordination period. When an interchange exchange amount is determined, it holds for all the real-time market periods within the coordination period. Hence, as the coordination period duration increases, it becomes more difficult to capture the prices at the real-time market periods within the coordination period. We run a simulation using actual real-time market prices in Sandy Pond and Roseton busses. The simulation results show that the number of real-time market periods during which interface exchange is counter-intuitive increases as the coordination period duration increases. Hence, it makes sense for the IGOs of the interconnected power systems to focus on the limiting constraints on the duration of the procedural steps of CTS, and strive to shorten the coordination periods.

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APPENDIX A

NOTATION

For any given day, all time elements are expressed in minutes (min). An element τ denotes the cumulative minutes elapsed from the start of the day at 0 min.

h	index of each hour of a day, $h = 1, 2, ...24$
$[k]\big\|_h$	index of an intra-hour subperiod of hour h, $k = 1, 2, ...K$

$\mathcal{T}_h \triangleq \{\tau : (h-1)(60) < \tau \leq (h)(60)\}$

ζ duration of each intra-hour subperiod $[k]\big|_h$

$\mathcal{T}_{[k]\big|_h} \triangleq \{\tau : (h-1)(60) + (k-1)(\zeta) < \tau \leq (h-1)(60) + (k)(\zeta)\}.$

All transactions across the interface are from/to † to/from ‡

i index of internal offers submitted to † , $i = 1, 2, ..., I$

j index of internal offers submitted to ‡, $j = 1, 2, ..., J$

n index of interface offers from † to ‡, $n = 1, 2, ..., N$

c index of interface offers from ‡ to †, $c = 1, 2, ..., C$

$\tilde{i}$ index of a modified internal offer at † created fictitiously by CTS, $\tilde{i} = 1, 2, ..., \tilde{I}$

$*$ the superscript that denotes the optimum of an optimization problem

$\acute{}$ the accent that associates a variable to the ED problem solved without any utilization of the interface

$\tilde{}$ the accent that associates a variable to the ED problem solved by the modification of the internal offers at †

$\breve{}$ the accent that indicates that an offer is cleared

$\circ$ the accent that associates a variable to TLO

a the optimization variable, represents the cleared real power in MW from an internal offer

χ	the dual variable of the equality constraint of the optimization problems in the write-up, implies the marginal price to serve an additional MW of demand
f_ℓ	interface exchange
$\dagger$	the bus of IGO^A
$\ddagger$	the bus of IGO^B
$\beta = \{p, \mu\}$	the couplet that represents the offer to sell p MW at μ $\$/MWh$
$d^\dagger$ ($d^\ddagger$)	the actual intra-hour demand at $\dagger$ ($\ddagger$)
$\bar{d}^\dagger$ ($\bar{d}^\ddagger$)	the forecasted intra-hour demand at $\dagger$ ($\ddagger$)

APPENDIX B

TIE-LINE OPTIMIZATION (TLO)

In this appendix, we describe tie-line optimization (TLO) that is deployed for the interchange evaluation of the two interconnected IGOs. As seen in Chapter 3, the framework developed in Chapter 2 was a comprehensive and general framework that can be applied to various interchange evaluation methodologies. In this appendix, we describe the TLO procedure by using the framework that we developed in Chapter 2. In our description of TLO, we utilize the time scale that we introduced in Chapter 2.1, and describe the performance of TLO to evaluate the interchange for the intra-hourly subperiod $[k]\big|_h$.

In Section B.1, we tailor the framework described in Chapter 2 for TLO. We devote Section B.2 to the description of TLO. Finally, we present our concluding remarks on the appendix in Section B.3.

B.1 The TLO Framework

In Chapter 2, we have presented a general and comprehensive framework that could be used for the analysis and the performance evaluation of any interchange evaluation methodology, and we have used the framework to represent CTS in Chapter 3. In this section, we tailor the framework in Chapter 2 for TLO and provide the specific models that get accommodated in the framework.

In order to accommodate the explicit representations of the physical aspects, economic aspects, the interactions between the physical and economic aspects, and the coordination procedure, we have constructed the framework to consist of two interconnected layers: the physical layer and the economic layer. The physical layer of the framework accommodated the power system models and the interface. TLO assumes the hypothetical aggregation of two interconnected systems, and that the hypothetically aggregated system is operated by a hypothetical super-IGO that takes over the functions of both $IGOs$ and so has the knowledge of the offers and the forecasted intra-hour demands of the two $IGOs$. Hence, TLO recognizes neither the identities nor the independence of the two interconnected $IGOs$, and treats the interface as an internal line of the hypothetically aggregated IGO, which renders TLO an infeasible conceptual construct.

We model the power system of the super-IGO to consist of two busses † and †. In addition, the power system model of the super-IGO contains the information on the generators in the aggregated system.

The two busses are physically connected via tie-line(s). There may be one or more physical tie-lines between the two busses, and the power flows between the two busses through the tie-line(s). For the purposes of interchange evaluation, we model the tie-lines as interface. In our model, the interface connects the bus † to the bus †. We model any power flow from/to † to/from † via the interface. We denote the real power flow on the interface by f_ℓ, and use the convention that if the power flow is from ‡ to †, then $f_\ell > 0$ and if the power flow is from † to ‡, then $f_\ell < 0$.

The other layer in the framework constructed in Chapter 2 is the economic layer, which accommodates all the economic aspects of interchange evaluation and the coordination procedure. We introduce the internal market model, which is the mechanism by which the super-IGO performs the wholesale purchase/sale of electricity of the aggregated system. We consider that the super-IGO internal market model has a uniform-price single-sided auction mechanism. The internal intra-hour market players of the super-IGO sell energy directly to the IGO by submitting sealed internal offers. Each such internal offer i (j)—where $i, i = 1, 2, ..., I$—that is submitted to the bus † is represented by the couplet $\beta_i^{\dagger} = \{p_i^{\dagger}, \mu_i^{\dagger}\}$, where $p_i^{\dagger}$ is the offered active power in MW and $\mu_i^{\dagger}$ $\$/MWh$ is the offer price. Similarly, each such internal offer j—where $j, j = 1, 2, ..., J$—that is submitted to the bus ‡ is represented by the couplet $\beta_j^{\ddagger} = \{p_j^{\ddagger}, \mu_j^{\ddagger}\}$, where $p_j^{\ddagger}$ is the offered active power in MW and $\mu_j^{\ddagger}$ $\$/MWh$ is the offer price.

We represent the set of all internal offers submitted at † (‡) by $\{\beta^{\dagger}\}$ ($\{\beta^{\ddagger}\}$).

In our analysis of the intra-hourly internal markets, we consider the intra-hour demand in the market model of the super-IGO to be price-insensitive. We refer to the intra-hour demand of each IGO as the residual demand that is not cleared in the day-ahead market and that needs to be cleared during the intra-hourly subperiod $[k]\big|_h$. We represent the intra-hour demand at the bus † (‡) with $d^{\dagger}$ ($d^{\ddagger}$). The economic layer also accommodates the TLO procedure that the super-IGO executes for the determination of interchange evaluation.

The models that are described above get accommodated in their respective layers of the framework. Namely, super-IGO power system model, and the interface get accommodated in the physical layer, and the super-IGO internal market model and the TLO procedure get accommodated in the economic layer. In addition to these models, it is required to consider the interactions of these models with each other to execute TLO. The framework accommodates the interactions of these models with each other through the flow of data/information. We have observed in Chapter 3 that the share of data/information is a critical matter in CTS. However, since in TLO we assume the hypothetical aggregation of the two systems, we no longer have the problems associated with the confidentiality of data/information.

B.2 *TLO* Problem Formulation

The determination of the power flow amount on the interface by the TLO is a by-product of the solution of an ED problem for the hypothetically aggregated system using $\{\beta^{\dagger}\}$ and $\{\beta^{\ddagger}\}$ to meet the aggregated intra-hour demand $d^{\dagger} + d^{\ddagger}$ for a specified period. The ED function serves to allocate the submitted internal offers so as to minimize the payments to serve the aggregated intra-hour demand.

In order to formulate the ED problem, we introduce the variable $\mathring{a}_i^{\dagger}$ ($\mathring{a}_j^{\ddagger}$), which represents the cleared real power in MW from the i^{th} (j^{th}) submitted internal offer. Hence, $\mathring{a}_i^{\dagger}$, $i = 1, 2, ..., I$ ($\mathring{a}_j^{\ddagger}$, $j = 1, 2, ..., J$) serve as the **optimization variables** in the ED problem formulation.

The TLO problem for the fictitious aggregated system can be formulated as follows:

$$\begin{aligned}
&\min_{\mathring{a}_i^{\dagger}, \mathring{a}_j^{\ddagger}} \quad \sum_{i=1}^{I} (\mathring{a}_i^{\dagger})\,(\mu_i^{\dagger}) \; + \; \sum_{j=1}^{J} (\mathring{a}_j^{\ddagger})\,(\mu_j^{\ddagger}) \\
&s.t. \quad \sum_{i=1}^{I} \mathring{a}_i + \sum_{j=1}^{J} \mathring{a}_j \; - \; (\bar{d}^{\dagger} + \bar{d}^{\ddagger}) \; = \; 0 \quad \longleftrightarrow \quad \mathring{\chi} \\
&\qquad \mathring{a}_i - p_i \; \leq 0 \quad i \; = \; 1, 2, ...I \\
&\qquad \mathring{a}_j - p_j \; \leq 0 \quad j \; = \; 1, 2, ...J
\end{aligned} \tag{B.1}$$

The optimal clearance variables $\mathring{a}_i^{\dagger *}$ $i = 1, 2, ...I$ and $\mathring{a}_j^{\ddagger *}$ indicate the MW of power cleared from the internal offers $i = 1, 2, ...I$ and $j = 1, 2, ...J$. Further, we denote the dual variable associated with the equality constraint at the optimum as $\mathring{\chi}^*$. $\mathring{\chi}^*$ is the marginal price to serve the last MW of the aggregated demand $\bar{d}^{\dagger} + \bar{d}^{\ddagger}$.

The objective function value obtained with the optimal decision variables corresponds to the total payments obtained via TLO, which is $\sum_{i=1}^{I}(\mathring{a}_i^{\dagger *})(\mu_i^{\dagger}) + \sum_{j=1}^{J}(\mathring{a}_j^{\ddagger *})(\mu_j^{\ddagger})$.

Furthermore, the total amount of power sold by the offerers at $\dagger$ can be represented by $\sum_{i=1}^{I} \mathring{a}_i^{\dagger}$. Similarly, the total amount of power sold by the internal offerers at $\ddagger$ can be represented as $\sum_{j=1}^{J} \mathring{a}_j^{\ddagger}$. Therefore, we can write the following equations:

$$\begin{aligned} \sum_{j=1}^{J} \mathring{a}_j^{\ddagger} &= d^{\ddagger} + f_{\ell} \\ \sum_{i=1}^{I} \mathring{a}_i^{\dagger} + f_{\ell} &= d^{\dagger} \end{aligned} \tag{B.2}$$

which will allow us to obtain $f_{\ell} = 45\,MW$.

B.3 Concluding Remarks and Summary

In this appendix, we have presented the TLO procedure. We have specifically tailored the framework developed in Chapter 2 for TLO. We have described the condition that TLO assumes the hypothetical aggregation of the two systems, and that the hypothetically aggregated system is operated by a super-IGO. We have also provided the mathematical formulation of TLO, and derived the mathematical expressions for the total payments and the interface flow. Since the TLO is an ideal construct that treats the interface as an internal line of the super-IGO, the total payments obtained via TLO can be used as a benchmark with which the total payment of other coordination schemes can be compared.